Supplementary Notes to

## IIT JEE Physics

## Topic-wise Complete Solutions

Vernier Calipers Made Easy for
Physics Students and Teachers


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## Read Me First!

These notes are to supplement your regular textbooks, not to replace them. We made an attempt to make Physics easy and interesting.
To make best use of these notes, you should get familiar with the Vernier calipers. Experiment with the Vernier calipers available in Physics laboratory or buy your own. It does not cost much!

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We would be glad to hear your suggestions for the improvement of these notes. If you find any conceptual errors or typographical errors, howsoever small and insignificant, please inform us so that these can be corrected.
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## 1 How much we know about the Vernier calipers?

Measurement is a fundamental part of all scientific experiments, including Physics. At one extreme, our vast universe extends this measuring exercise to light-years, the distances so vast that we cannot see them from our eyes. On the other extreme of the smallest distances, new discoveries are pushing it down to femto-meter $\left(10^{-15} \mathrm{~m}\right)$ or even less. These distances are so small that we cannot see them from our eyes. At every $1-2$ order of magnitude change in distance, our instruments to measure the distances accurately can differ. When the distances, we want to measure, are in the range of $10^{-2} \mathrm{~mm}$ to 1 mm , we use Vernier Calipers and Screw Gauge for precise measurement. In this article, we are going to focus on these measuring instruments only.

In class $11^{\text {th }}$ Physics lab, we were trained to answer the following questions:
(a). How to find the Least Count (LC) or Vernier Constant?
(b). How to read Main Scale Reading (MSR) and Vernier Scale Reading (VSR)?
(c). How to find the zero error?
(d). How to use the above data to get the final measurement?

Answers to these how to questions kept us content for 22 years. But this was not adequate to solve the IIT JEE 2016 problem. We needed to figure out some more interesting why questions like:
(A). Why least count is given by

$$
\mathrm{LC}=\frac{\text { Value of } 1 \text { main scale division }}{\text { Total number of divisions on Vernier scale }}
$$

(B). Why the measured value is given by

$$
\text { Observed Value }=\mathrm{MSR}+\mathrm{LC} \times \mathrm{VSR}
$$

(C). Why the zero error is subtracted from the observed value i.e.,

$$
\text { True Value }=\text { Observed Value }- \text { Zero Error }
$$

Let us start our journey with a Vernier calipers without zero error. When two jaws are closed, $0^{\text {th }}$ mark on the Vernier scale is aligned with the $0^{\text {th }}$ mark on the main scale as shown in the figure 1. Also note that $10^{\text {th }}$ mark on Vernier scale coincides with the $9^{\text {th }}$ mark on main scale.

One main scale division (MSD) is the distance between two successive marks on the main scale. It is given in the figure 1 that 1 MSD is equal to 1 mm . One Vernier scale division (VSD) is the distance between two successive marks on the Vernier scale. It is given that $10 \mathrm{VSD}=9 \mathrm{MSD}$. Thus, $1 \mathrm{VSD}=(9 / 10) \mathrm{MSD}=(9 / 10) 1 \mathrm{~mm}=0.9 \mathrm{~mm}$ i.e., distance between two successive marks on the Vernier scale is 0.9 mm .


Figure 1: Vernier calipers with no zero error

Now, let us use this Vernier calipers to measure the diameter $D$ of a marble ball (marble balls usually have $D=1 / 2 \mathrm{in}$ ). The measurement is shown in the figure 2. Here, $x_{m 0}$ is the distance between the jaw attached to main scale (left jaw) and the $0^{\text {th }}$ mark on the main scale and $x_{v 0}$ is
the distance between the jaw attached to Vernier scale (right jaw) and the $0^{\text {th }}$ mark on the Vernier scale. Observe that the diameter is given by

$$
\begin{equation*}
D=x_{m 0}+x \tag{1}
\end{equation*}
$$

where $x$ is the distance between the $0^{\text {th }}$ mark on the main scale and the right jaw.
Now, note that the $7^{\text {th }}$ mark on Vernier scale coincides with 1.9 cm on the main scale. This point is called the point of coincidence. The distance between the $0^{\text {th }}$ mark on main scale and the point of coincidence is $x_{m}$ and the distance between the $0^{\text {th }}$ mark on Vernier scale and the point of coincidence is $x_{v}$. Thus,

$$
\begin{equation*}
x+x_{v 0}+x_{v}=x_{m} \tag{2}
\end{equation*}
$$

Substitute $x$ from equation 2 into equation 1 to get

$$
\begin{equation*}
D=\left(x_{m}-x_{v}\right)-\left(x_{m 0}-x_{v 0}\right) \tag{3}
\end{equation*}
$$

The quantity $\left(x_{m 0}-x_{v 0}\right)$ is called the zero error of the Vernier calipers. Negative of the zero error is called zero correction. Note that $x_{m 0}=x_{v 0}$ in a Vernier calipers without zero error (which is true in this case). Also, $x_{m}=19 \mathrm{MSD}=19 \mathrm{~mm}$ and $x_{v}=7 \mathrm{VSD}=7(9 / 10)=6.3 \mathrm{~mm}$. Substitute these values in equation 3 to get $D=12.7 \mathrm{~mm}=1.27 \mathrm{~cm}$ (half inch marble).


Figure 2: Measuring Diameter of a Marble Ball
There is another easier way to get the measured value. The main scale reading (MSR) is the first reading on the main scale immediately to the left of the zero of Vernier scale (MSR $=12 \mathrm{~mm}$ in this example). The Vernier scale reading (VSR) is the mark on Vernier scale which exactly coincides with a mark on the main scale (VSR $=7$ in this example). Note that there are VSR divisions on the main scale between MSR mark (i.e., mark on the main scale immediately to the left of the zero of the Vernier scale) and the point of coincidence. Thus,

$$
\begin{align*}
& x_{v}=\mathrm{VSR} \times \mathrm{VSD}  \tag{4}\\
& x_{m}=\mathrm{MSR}+\mathrm{VSR} \times \mathrm{MSD} \tag{5}
\end{align*}
$$

Corresponding values of the parameters for the zero errors are

$$
\begin{align*}
& x_{v 0}=\mathrm{VSR}_{0} \times \mathrm{VSD}  \tag{6}\\
& x_{m 0}=\mathrm{MSR}_{0}+\mathrm{VSR}_{0} \times \mathrm{MSD} \tag{7}
\end{align*}
$$

Substitute in equation 3 to get

$$
\begin{align*}
D= & ((\mathrm{MSR}+\mathrm{VSR} \times \mathrm{MSD})-(\mathrm{VSR} \times \mathrm{VSD}))- \\
& \left(\left(\mathrm{MSR}_{0}+\mathrm{VSR}_{0} \times \mathrm{MSD}\right)-\left(\mathrm{VSR}_{0} \times \mathrm{VSD}\right)\right) \\
= & (\mathrm{MSR}+\mathrm{VSR}(\mathrm{MSD}-\mathrm{VSD}))-\left(\mathrm{MSR}_{0}+\mathrm{VSR}_{0}(\mathrm{MSD}-\mathrm{VSD})\right) \\
= & (\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LC})-\left(\mathrm{MSR}_{0}+\mathrm{VSR}_{0} \times \mathrm{LC}\right) \\
= & \mathrm{MSR}+\mathrm{VSR} \times \mathrm{LC}-\text { Zero Error. } \tag{8}
\end{align*}
$$

where $\mathrm{LC}=\mathrm{MSD}-\mathrm{VSD}$ is called the least count or Vernier constant. It is the smallest length that can be measured accurately with a Vernier calipers. For the given Vernier calipers

$$
\begin{aligned}
& \mathrm{LC}=\mathrm{MSD}-\mathrm{VSD}=1 \mathrm{~mm}-9 / 10 \mathrm{~mm}=0.1 \mathrm{~mm} \\
& D=\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LC}=12 \mathrm{~mm}+7 \times(0.1)=12.7 \mathrm{~mm}=1.27 \mathrm{~cm}
\end{aligned}
$$

Note that Vernier calipers can be used to measure (1) outer dimensions like diameter of a sphere or edge of a cube (2) inner dimensions like inner diameter of a hollow cylinder and (3) depth of a hollow cylinder.

## 2 Worked Out Examples

Example 1: The jaws of the Vernier calipers shown in figure 3 are in contact with each other. Find the zero error of this Vernier calipers.


Figure 3: Vernier calipers with positive zero error

Solution: The least count of given Vernier calipers is

$$
\mathrm{LC}=\mathrm{MSD}-\mathrm{VSD}=1-(9 / 10)=0.1 \mathrm{~mm} .
$$

The main scale reading is $\mathrm{MSR}_{0}=0 \mathrm{~mm}$ and the Vernier scale reading is $\mathrm{VSR}_{0}=3$. Thus,

$$
\text { Zero Error }=\mathrm{MSR}_{0}+\mathrm{VSR}_{0} \times \mathrm{LC}=0+3 \times 0.1=0.3 \mathrm{~mm} .
$$

Example 2: The Vernier calipers of example 1 is used to measure the edge of a cube. The readings are shown in the figure 4 . Find the edge length of the cube.


Figure 4: Measuring the edge of a cube

Solution: The readings are $\operatorname{MSR}=25 \mathrm{~mm}$ and $\mathrm{VSR}=7$. Thus,

$$
\begin{aligned}
a & =\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LS}-\text { Zero Error } \\
& =25+7(0.1)-0.3=25.4 \mathrm{~mm}=2.54 \mathrm{~cm} .
\end{aligned}
$$

Example 3: The jaws of the Vernier calipers, shown in figure 5, are in contact with each other.
Find the zero error of this Vernier calipers.


Figure 5: Vernier calipers with negative zero error

Solution: This is an interesting problem. What is $\mathrm{MSR}_{0}$ ? It is the first reading on the main scale immediately to the left of the zero of the Vernier scale. But there are no marks on the main scale before zero of the Vernier scale. We claim that $\mathrm{MSR}_{0}=-1 \mathrm{~mm}$ (observe it carefully, why $\mathrm{MSR}_{0}$ is not equal to -2 mm ?). The Vernier scale reading is $\mathrm{VSR}_{0}=4$ and the least count is $\mathrm{LC}=0.1 \mathrm{~mm}$. Substitute these values to get,

$$
\text { Zero Error }=\mathrm{MSR}_{0}+\mathrm{VSR}_{0} \times \mathrm{LC}=-1+4 \times 0.1=-0.6 \mathrm{~mm}
$$

Example 4: The Vernier calipers of example 3 is used to measure the edge of a cube. The readings are shown in the figure 6 Find the edge length of the cube.


Figure 6: Measuring edge length of a cube

Solution: The readings are $\mathrm{MSR}=24 \mathrm{~mm}$ and $\mathrm{VSR}=8$. Thus,

$$
\begin{aligned}
a & =\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LS}-\text { Zero Error } \\
& =24+8(0.1)-(-0.6)=25.4 \mathrm{~mm}=2.54 \mathrm{~cm} .
\end{aligned}
$$

Example 5: What is the LC of the Vernier calipers shown in the figure 7?
Solution: One main scale division is $1 \mathrm{MSD}=1 \mathrm{~mm}$. Since $5 \mathrm{VSD}=4 \mathrm{MSD}$, we get $1 \mathrm{VSD}=$ $(4 / 5) \mathrm{MSD}=0.8 \mathrm{~mm}$. Thus, the least count of this calipers is

$$
\mathrm{LC}=\mathrm{MSD}-\mathrm{VSD}=1.0-0.8=0.2 \mathrm{~mm} .
$$



Figure 7: Least Count of a Vernier callipers

## 3 IIT JEE Solved Problems

Question. $N$ divisions on the main scale of a Vernier calipers coincide with $(N+1)$ divisions on its Vernier scale. If each division on the main scale is of $a$ units, determine the least count of instrument.
Solution. Given, a main scale division (MSD) of the Vernier calipers is

$$
\begin{equation*}
1 \mathrm{MSD}=a \tag{1}
\end{equation*}
$$

Since $(N+1)$ Vernier scale divisions (VSD) are equal to $N$ main scale divisions, we get

$$
\begin{equation*}
1 \mathrm{VSD}=\frac{N}{N+1} \mathrm{MSD}=\frac{N a}{N+1} \tag{2}
\end{equation*}
$$

The least count is given by

$$
\mathrm{LC}=1 \mathrm{MSD}-1 \mathrm{VSD}=\frac{a}{N+1}
$$

Question. The edge of a cube is measured using a Vernier calipers (9 divisions of the main scale are equal to 10 divisions of Vernier scale and 1 main scale division is 1 mm ). The main scale division reading is 10 and first division of Vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g . Calculate the density in $\mathrm{g} / \mathrm{cm}^{3}$ up to correct significant figures.
Solution. From the given data, one main scale division (MSD) is

$$
\begin{equation*}
1 \mathrm{MSD}=1 \mathrm{~mm} \tag{1}
\end{equation*}
$$

Since 10 Vernier scale divisions (VSD) are equal to 9 MSD , we get

$$
\begin{equation*}
1 \mathrm{VSD}=9 / 10=0.9 \mathrm{~mm} \tag{2}
\end{equation*}
$$

The least count (LC) is given by

$$
\begin{equation*}
\mathrm{LC}=1 \mathrm{MSD}-1 \mathrm{VSD}=1.0-0.9=0.1 \mathrm{~mm} \tag{3}
\end{equation*}
$$

Given, the main scale reading (MSR) is 10 and the Vernier scale reading (VSR) is 1 . The measured value of the edge is given by

$$
a=\mathrm{MSR} \times \mathrm{MSD}+\mathrm{VSR} \times \mathrm{LC}=10 \times 1+1 \times 0.1=10.1 \mathrm{~mm}
$$

The measurement of $a$ has three significant digits. The volume of the cube is $V=a^{3}=1.03 \mathrm{~cm}^{3}$ and the density is

$$
m / V=2.736 / 1.03=2.6563=2.66 \mathrm{~g} / \mathrm{cm}^{3}
$$

(after rounding off to three significant digits).

Question. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
(2010)
(A). 0.02 mm
(B). 0.05 mm
(C). 0.1 mm
(D). 0.2 mm

Solution. For the given Vernier calipers, one main scale division (MSD) is

$$
\begin{equation*}
1 \mathrm{MSD}=1 \mathrm{~mm} \tag{1}
\end{equation*}
$$

Since 20 Vernier scale divisions (VSD) are equal to 16 MSD , we get

$$
\begin{equation*}
1 \mathrm{VSD}=16 / 20=0.8 \mathrm{~mm} . \tag{2}
\end{equation*}
$$

The least count (LC) is given by

$$
\mathrm{LC}=1 \mathrm{MSD}-1 \mathrm{VSD}=1.0-0.8=0.2 \mathrm{~mm} .
$$

Question. The diameter of a cylinder is measured using a Vernier calipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm . The $24^{\text {th }}$ division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is
(2013)
(A). 5.112 cm
(B). 5.124 cm
(C). 5.136 cm
(D). 5.148 cm

Solution. From the given data, one main scale division (MSD) and one Vernier scale division (VSD) are

$$
\begin{align*}
& 1 \mathrm{MSD}=5.15 \mathrm{~cm}-5.10 \mathrm{~cm}=0.05 \mathrm{~cm}  \tag{1}\\
& 1 \mathrm{VSD}=\frac{2.45}{50} \mathrm{~cm}=0.049 \mathrm{~cm} \tag{2}
\end{align*}
$$

The least count (LC) of the given Vernier calipers is

$$
\begin{equation*}
\mathrm{LC}=1 \mathrm{MSD}-1 \mathrm{VSD}=0.001 \mathrm{~cm} \tag{3}
\end{equation*}
$$

For the given measurement, main scale reading (MSR) is 5.10 cm and the Vernier scale reading (VSR) is 24 . Hence, the diameter $D$ of the cylinder is

$$
D=\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LC}=5.10+24 \times 0.001=5.124 \mathrm{~cm}
$$

Question. Consider a Vernier calipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier calipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then,
(2015)
(A). If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm .
(B). If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm .
(C). If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm .
(D). If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm .

Solution. In given Vernier calipers, each 1 cm is equally divided into 8 main scale divisions (MSD). Thus, $1 \mathrm{MSD}=1 / 8=0.125 \mathrm{~cm}$. Further, 4 main scale divisions coincide with 5 Vernier scale divisions (VSD) i.e., $4 \mathrm{MSD}=5 \mathrm{VSD}$. Thus, $1 \mathrm{VSD}=4 / 5 \mathrm{MSD}=0.8 \times 0.125=0.1 \mathrm{~cm}$. The least count of the Vernier calipers is given by

$$
\begin{equation*}
\mathrm{LC}=1 \mathrm{MSD}-1 \mathrm{VSD}=0.125-0.1=0.025 \mathrm{~cm} . \tag{1}
\end{equation*}
$$

In screw gauge, let $l$ be the distance between two adjacent divisions on the linear scale. The pitch $p$ of the screw gauge is the distance travelled on the linear scale when it makes one complete rotation. Since circular scale moves by two divisions on the linear scale when it makes one complete rotation, we get $p=2 l$. The least count of the screw gauge is defined as ratio of the pitch to the number of divisions on the circular scale ( $n$ ) i.e.,

$$
\begin{equation*}
\mathrm{lc}=p / n=2 l / 100=l / 50 \tag{2}
\end{equation*}
$$

If $p=2 \mathrm{LC}=2(0.025)=0.05 \mathrm{~cm}$, then $l=p / 2=0.025 \mathrm{~cm}$. Substitute $l$ in equation 2 to get the least count of the screw gauge

$$
\begin{equation*}
\mathrm{lc}=0.025 / 50=5 \times 10^{-4} \mathrm{~cm}=0.005 \mathrm{~mm} \tag{3}
\end{equation*}
$$

If $l=2 \mathrm{LC}=2(0.025)=0.05 \mathrm{~cm}$ then equation 2 gives

$$
\begin{equation*}
\mathrm{lc}=0.05 / 50=1 \times 10^{-3} \mathrm{~cm}=0.01 \mathrm{~mm} \tag{4}
\end{equation*}
$$

Question. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers $\left(C_{1}\right)$ has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper $\left(C_{2}\right)$ has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm ) by calipers $C_{1}$ and $C_{2}$, respectively are


(A). 2.85 and 2.82
(B). 2.87 and 2.83
(C). 2.87 and 2.86
(D). 2.87 and 2.87

Solution. In both calipers $C_{1}$ and $C_{2}, 1 \mathrm{~cm}$ is divided into 10 equal divisions on the main scale. Thus, 1 division on the main scale is equal to $x_{m 1}=x_{m 2}=1 \mathrm{~cm} / 10=0.1 \mathrm{~cm}$. In calipers $C_{1}, 10$ equal divisions on the Vernier scale are equal to 9 main scale divisions. Thus, 1 division on the Vernier scale of $C_{1}$ is equal to $x_{v 1}=9 x_{m 1} / 10=0.09 \mathrm{~cm}$. In calipers $C_{2}, 10$ equal divisions on the Vernier scale are equal to 11 main scale divisions. Thus, 1 division on the Vernier scale of $C_{2}$ is equal to $x_{v 2}=11 x_{m 2} / 10=0.11 \mathrm{~cm}$.


Let main scale reading be MSR and $v^{\text {th }}$ division of the Vernier scale coincides with $m^{\text {th }}$ division of the main scale ( $m$ is counted beyond MSR). The value measured by this calipers is

$$
\begin{equation*}
X=\mathrm{MSR}+x=\mathrm{MSR}+m x_{m}-v x_{v} \tag{1}
\end{equation*}
$$

In calipers $C_{1}, \mathrm{MSR}_{1}=2.8 \mathrm{~cm}, m_{1}=7$ and $v_{1}=7$ and in calipers $C_{2}, \mathrm{MSR}_{2}=2.8 \mathrm{~cm}, m_{2}=8$ and $v_{2}=7$. Substitute these values in equation 1 to get

$$
\begin{aligned}
& X_{1}=\mathrm{MSR}_{1}+m_{1} x_{m 1}-v_{1} x_{v 1}=2.8+7(0.1)-7(0.09)=2.87 \mathrm{~cm} \\
& X_{2}=\operatorname{MSR}_{2}+m_{2} x_{m 2}-v_{2} x_{v 2}=2.8+8(0.1)-7(0.11)=2.83 \mathrm{~cm}
\end{aligned}
$$

The Vernier calipers are generally of type $C_{1}$ having $m=v$ and least count $\mathrm{LC}=x_{m}-x_{v}$. For these calipers, equation 1 gives $X=\mathrm{MSR}+v \times \mathrm{LC}$.
Question. During Searle's experiment, zero of the Vernier scale lies between $3.20 \times 10^{-2} \mathrm{~m}$ and $3.25 \times 10^{-2} \mathrm{~m}$ of the main scale. The $20^{\text {th }}$ division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between $3.20 \times 10^{-2} \mathrm{~m}$ and $3.25 \times 10^{-2} \mathrm{~m}$ of the main scale but now $45^{\text {th }}$ division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7} \mathrm{~m}^{2}$. The least count of the Vernier scale is $1.0 \times 10^{-5} \mathrm{~m}$. The maximum percentage error in the Young's modulus of the wire is $\qquad$
(2014)

Solution. The difference between the two measurements by Vernier scale gives elongation of the wire caused by the additional load of 2 kg . In the first measurement, main scale reading is $\mathrm{MSR}=3.20 \times 10^{-2} \mathrm{~m}$ and Vernier scale reading is VSR $=20$. The least count of Vernier scale is $\mathrm{LC}=1 \times 10^{-5} \mathrm{~m}$. Thus, the first measurement by Vernier scale is

$$
\begin{align*}
L_{1} & =\mathrm{MSR}+\mathrm{VSR} \times \mathrm{LC} \\
& =3.20 \times 10^{-2}+20\left(1 \times 10^{-5}\right)=3.220 \times 10^{-2} \mathrm{~m} \tag{1}
\end{align*}
$$

In the second measurement, $\mathrm{MSR}=3.20 \times 10^{-2} \mathrm{~m}$ and $\mathrm{VSR}=45$. Thus, the second measurement by Vernier scale is

$$
\begin{equation*}
L_{2}=3.20 \times 10^{-2}+45\left(1 \times 10^{-5}\right)=3.245 \times 10^{-2} \mathrm{~m} \tag{2}
\end{equation*}
$$

The elongation of the wire due to force $F=2 g$ is

$$
\begin{equation*}
l=L_{2}-L_{1}=0.025 \times 10^{-2} \mathrm{~m} \tag{3}
\end{equation*}
$$

The maximum error in the measurement of $l$ is $\Delta l=\mathrm{LC}=1 \times 10^{-5} \mathrm{~m}$. Young's modulus is given by $Y=\frac{F L}{l A}$. The maximum percentage error in the measurement of $Y$ is

$$
\frac{\Delta Y}{Y} \times 100=\frac{\Delta l}{l} \times 100=\frac{1 \times 10^{-5}}{0.025 \times 10^{-2}} \times 100=4 \%
$$

The readers are encouraged to compute the values of $Y$ and $\Delta Y$. Hint: $Y=2 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$ and $\Delta Y=0.08 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$.

## 4 Exercise Problems

Problem 1. The jaws of the Vernier calipers shown in figure 8 are in contact with each other. Find the zero error of this Vernier calipers.

Ans. 1.9 mm


Figure 8: Zero error of the Vernier calipers

Problem 2. The jaws of the Vernier calipers shown in figure 9 are in contact with each other. Find the zero error of this Vernier calipers.

Ans. -1.2 mm


Figure 9: Zero error of the Vernier calipers

Problem 3. The zero error of the Vernier calipers shown in figure 10 is 0.9 mm . What is the diameter of the sphere being measured in figure 10 .

Ans. 3.14 cm


Figure 10: Measuring diameter of a sphere with an erroneous Vernier calipers

Problem 4. The zero error of the Vernier calipers shown in figure 11 is -0.5 mm . What is the diameter of the sphere being measured in figure 11.

Ans. 3.14 cm
Problem 5. Choose the wrong statement for zero error and zero correction


Figure 11: Measuring diameter of a sphere with an erroneous Vernier calipers
(A). If the zero of the Vernier scale does not coincide with the zero of the main scale then the instrument is said to be having a zero error.
(B). Zero error is positive when the zero of Vernier scale lies to the left of the zero of the main scale.
(C). Zero correction has a magnitude equal to zero error but sign is opposite to that of the zero error.
(D). None of the above is wrong.

Ans. (B)
Problem 6. What is Vernier constant?
(A). It is the value of one main scale division divided by the total number of divisions on the main scale.
(B). It is the value of one Vernier scale division divided by the total number of divisions on the Vernier scale.
(C). It is the difference between value of one main scale division and one Vernier scale division.
(D). It is also the least count of the Vernier scale.

Ans. (C), (D)
Problem 7. The smallest division on main scale of a Vernier calipers is 1 mm and 10 Vernier scale division coincide with 9 main scale divisions. While measuring the length of a line, the zero mark of the Vernier scale lies between 10.2 cm and 10.3 cm and the third division of Vernier scale coincide with a main scale division. (a) Determine the least count of the calipers, and (b) Length of the line.

Ans. $0.01 \mathrm{~cm}, 10.23 \mathrm{~cm}$

Problem 8. The main scale of a Vernier calipers is calibrated in mm and 19 divisions of main scale are equal in length to 20 divisions of Vernier scale. In measuring the diameter of a cylinder by this instrument, the main scale reads 35 divisions and 4th division of Vernier scale coincides with a main scale division. Find (a) least count of the Vernier calipers and (b) radius of the cylinder.

Ans. $0.005 \mathrm{~cm}, 1.76 \mathrm{~cm}$
Problem 9. Least count of a Vernier calipers is 0.01 cm . When the two jaws of the instrument touch each other the $5^{\text {th }}$ division of the Vernier scale coincide with a main scale division and the zero of the Vernier scale lies to the left of the zero of the main scale. Furthermore while measuring the diameter of a sphere, the zero mark of the Vernier scale lies between 2.4 cm and 2.5 cm and the $6^{\text {th }}$ Vernier division coincides with a main scale division. Calculate the diameter of the sphere. Ans. 2.51 cm

Problem 10. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the Vernier scale. If the smallest division of the main scale is half-a-degree $\left(=0.5^{\circ}\right)$ then the least count of the instrument is

AIEEE 2009
(A). one minute
(B). half minute
(C). one degree
(D). half degree

Ans. (A)
Problem 11. A student measured the length of a rod and wrote it as 3.50 cm . Which instrument did he use to measure it?
(A). A meter scale
(B). A Vernier caliper where the 10 divisions in Vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm
(C). A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(D). A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Ans. (B)
Problem 12. 19 divisions on the main scale of a Vernier calipers coincide with 20 divisions on the Vernier scale. If each division on the main scale is of 1 cm , determine the least count of instrument.

Ans. 0.05 cm
Problem 13. The angle of a prism is measured by a spectrometer. The main scale reading is 58.5 degree and Vernier scale reading is 9 divisions. Given that 1 division on main scale corresponds to 0.5 degree and 30 divisions on the Vernier scale match with 29 divisions on the main scale. The angle of the prism from the above data is
(AIEEE 2012)
(A). 58.59 degree
(B). 58.77 degree
(C). 58.65 degree
(D). 59 degree

Ans. (C)
Problem 14. 1 cm on the main scale of a Vernier calipers is divided into 10 equal parts. If 10 divisions of Vernier coincides with 8 small divisions of main scale, then the least count of the calipers is
(A). 0.01 cm
(B). 0.05 cm
(C). 0.005 cm
(D). 0.02 cm

Ans. (D)
Problem 15. The jaws of a Vernier calipers touch the inner wall of calorimeter without any undue pressure. The position of zero of Vernier scale on the main scale reads 3.48 cm . The 6 th division of Vernier scale division coincides with a main scale division. Vernier constant of calipers is 0.01 cm . Find actual internal diameter of calorimeter, when it is observed that the Vernier scale has a zero error of -0.03 cm .
(A). 3.37 cm
(B). 3.57 cm
(C). 3.42 cm
(D). 3.54 cm

Ans. (B)
Problem 16. In a travelling microscope 1 cm on main scale is divided into 20 equal divisions and there are 50 divisions on the Vernier scale. What is the least count of microscope? Ans. 0.001 cm

Problem 17. The Vernier constant of a travelling microscope is 0.001 cm . If 49 main scale divisions coincide with 50 Vernier scale divisions, then the value of 1 main scale division is
(A). 0.1 mm
(B). 0.4 mm
(C). 0.5 mm
(D). 1 mm

Ans. (C)
Problem 18. The thin metallic strip of Vernier calipers moves downward from top to bottom in such a way that it just touches the surface of beaker. Main scale reading of caliper is 6.4 cm whereas its Vernier constant is 0.1 mm . The 4th division of Vernier scale coincides with a main scale division. The actual depth of beaker is (when zero of Vernier coincides with zero of main scale)
(A). 6.64 cm
(B). 6.42 cm
(C). 6.44 cm
(D). 6.13 cm

Ans. (C)
Problem 19. In an instrument, there are 25 divisions on the Vernier scale which coincides with 24 divisions of the main scale. 1 cm on main scale is divided into 20 equal parts. The least count of the instrument is
(A). 0.002 cm
(B). 0.05 cm
(C). 0.001 cm
(D). 0.02 cm

Ans. (A)
Problem 20. 1 cm of main scale of a Vernier calipers is divided into 10 divisions. If least count of the calipers is 0.005 cm , then the Vernier scale must have
(A). 10 divisions
(B). 20 divisions
(C). 25 divisions
(D). 50 divisions

Ans. (B)
Problem 21. In a Vernier calipers, there are 10 divisions on the Vernier scale and 1 cm on main scale is divided in 10 parts. While measuring a length, the zero of the Vernier scale lies just ahead of 1.8 cm mark and 4th division of Vernier scale coincides with a main scale division. The value of length is
(A). 1.804 cm
(B). 1.840 cm
(C). 1.800 cm
(D). None of these

Ans. (D)
Problem 22. Diameter of a steel ball is measured using a Vernier calipers which has divisions of 0.1 cm on its main scale (MS) and 10 divisions of its Vernier scale (VS) match 9 divisions on the main scale. Three such measurements for a ball are (1) MSR $=0.5 \mathrm{~cm}, \mathrm{VSD}=8$ (2) $\mathrm{MSR}=0.5 \mathrm{~cm}, \mathrm{VSD}=4(3) \mathrm{MSR}=0.5 \mathrm{~cm}, \mathrm{VSD}=6$. If the zero error is -0.03 cm , then mean corrected diameter is
(A). 0.53 cm
(B). 0.56 cm
(C). 0.59 cm
(D). 0.52 cm

Ans. (C)
Problem 23. Each division on the main scale is 1 mm . Which of the following Vernier scales gives Vernier constant equal to 0.01 mm ?
(A). 9 mm divided into 10 divisions
(B). 90 mm divided into 100 divisions
(C). 99 mm divided into 100 divisions
(D). 9 mm divided into 100 divisions

Ans. (C)
Problem 24. In a Vernier calipers 1 cm of the main scale is divided into 20 equal parts. 19 divisions of the main scale coincide with 20 divisions on the Vernier scale. Find the least count of the instrument.

Ans. 0.025 cm
Problem 25. In a vernier calipers, one main scale division is $x \mathrm{~cm}$ and $n$ divisions of the vernier scale coincide with $(n-1)$ divisions of the main scale. The least count (in cm ) of the calipers is (AMU PMT 2009)
(A). $\frac{(n-1)}{n} x$
(B). $\frac{n x}{n-1}$
(C). $\frac{x}{n}$
(D). $\frac{x}{n-1}$

Ans. (C)

## 5 More...

You can make a low cost Vernier calipers by drawing main scale and Vernier scale on strips of paper etc. The lines on main scale may be separated by 1 cm . To get lines on the Vernier scale, you can divide 9 divisions on main scale into 10 equal divisions (it is a test for your skills in geometry).

Take a scales of 30 cm length and another scale of 15 cm length. The bigger scale is main scale of your Vernier calipers and smaller scale is Vernier scale. Draw 30 parallel lines on main scale at 1 cm distance each. Draw 10 parallel lines on Vernier scale at 0.9 cm each. Your Vernier caliper is


Figure 12: Your Vernier calipers
ready (see figure 12). Take another scale (or any other rectangular piece) as support. The object, whose length is to measured, is placed between the support and Vernier scale.

A Vernier thruster is a rocket engine used on a spacecraft for fine adjustments to the velocity of a spacecraft. The name is derived from Vernier calipers (named after Pierre Vernier) which have a primary scale for gross measurements, and a secondary scale for fine measurements.

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