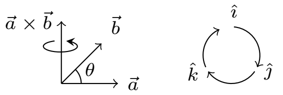
1 Vectors

Notation: $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:



$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a}\times\vec{b}| = ab\sin\theta$$

2 Kinematics

Average and Instantaneous Vel. and Accel.:

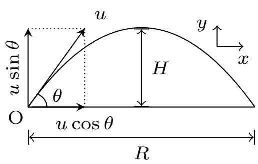
$$\vec{v}_{\rm av} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{\rm inst} = d\vec{r}/dt$$
$$\vec{a}_{\rm av} = \Delta\vec{v}/\Delta t \qquad \vec{a}_{\rm inst} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 - u^2 = 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:



$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

3 Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}, \quad \vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{\rm AB} = -\vec{F}_{\rm BA}$

Frictional force: $f_{\text{static, max}} = \mu_s N, \quad f_{\text{kinetic}} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta, \ \frac{v^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$

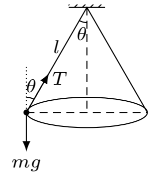
Centripetal force: $F_{\rm c} = \frac{mv^2}{r}, \quad a_{\rm c} = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\rm pseudo} = -m\vec{a}_0, \quad F_{\rm centrifugal} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\text{min, bottom}} = \sqrt{5gl}, \quad v_{\text{min, top}} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



4 Work, Power and Energy

Work: $W = \vec{F}\cdot\vec{S} = FS\cos\theta, \quad W = \int \vec{F}\cdot{\rm d}\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\rm gravitational} = mgh, \quad U_{\rm spring} = \tfrac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\rm conservative}\cdot{\rm d}\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conservative in nature.

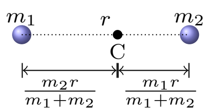
Power $P_{\rm av} = \frac{\Delta W}{\Delta t}, \quad P_{\rm inst} = \vec{F}\cdot\vec{v}$

5 Centre of Mass and Collision

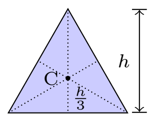
Centre of mass: $x_{\rm cm} = \frac{\sum x_i m_i}{\sum m_i}, \quad x_{\rm cm} = \frac{\int x{\rm d}m}{\int {\rm d}m}$

CM of few useful configurations:

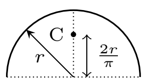
1. m_1, m_2 separated by r:



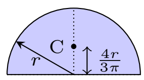
2. Triangle (CM $\equiv$ Centroid) $y_c = \frac{h}{3}$



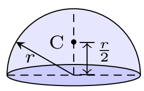
3. Semicircular ring: $y_c = \frac{2r}{\pi}$



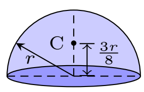
4. Semicircular disc: $y_c = \frac{4r}{3\pi}$



5. Hemispherical shell: $y_c = \frac{r}{2}$



6. Solid Hemisphere: $y_c = \frac{3r}{8}$



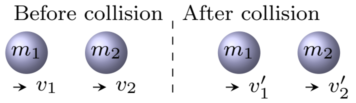
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}}, \quad \vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{ext}}}{M}$$

Impulse: $\vec{J} = \int \vec{F}\,\mathrm{d}t = \Delta\vec{p}$

Collision:



Momentum conservation: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
Elastic Collision: $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
Coefficient of restitution:

$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.

6 Rigid Body Dynamics

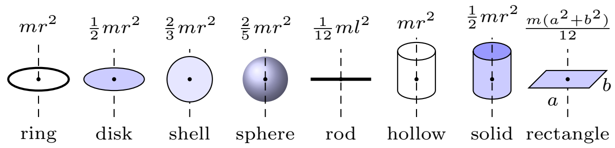
Angular velocity: $\omega_{\text{av}} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}, \quad \vec{v} = \vec{\omega} \times \vec{r}$

Angular Accel.: $\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}, \quad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}, \quad \vec{a} = \vec{\alpha} \times \vec{r}$

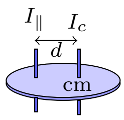
Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \tfrac{1}{2}\alpha t^2, \quad \omega^2 - {\omega_0}^2 = 2\alpha\theta$$

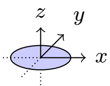
Moment of Inertia: $I = \sum_i m_i {r_i}^2, \quad I = \int r^2 \mathrm{d}m$



Theorem of Parallel Axes: $I_\parallel = I_{\text{cm}} + md^2$



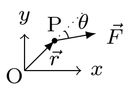
Theorem of Perp. Axes: $I_z = I_x + I_y$



Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}, \quad \vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}, \quad \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}, \quad \tau = I\alpha$



Conservation of $\vec{L}$: $\vec{\tau}_{\text{ext}} = 0 \implies \vec{L} = \text{const.}$

Equilibrium condition: $\sum \vec{F} = \vec{0}, \quad \sum \vec{\tau} = \vec{0}$

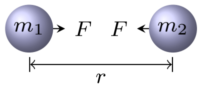
Kinetic Energy: $K_{\text{rot}} = \frac{1}{2}I\omega^2$

Dynamics:

$$\vec{\tau}_{\text{cm}} = I_{\text{cm}}\vec{\alpha}, \quad \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}, \quad \vec{p}_{\text{cm}} = m\vec{v}_{\text{cm}}$$
$$K = \tfrac{1}{2}m{v_{\text{cm}}}^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2, \quad \vec{L} = I_{\text{cm}}\vec{\omega} + \vec{r}_{\text{cm}} \times m\vec{v}_{\text{cm}}$$

7 Gravitation

Gravitational force: $F = G\frac{m_1m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

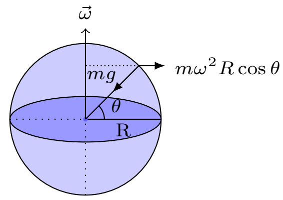
Variation of g with depth: $g_{\text{inside}} \approx g\left(1 - \frac{h}{R}\right)$

Variation of g with height: $g_{\text{outside}} \approx g\left(1 - \frac{2h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{\text{at pole}} > g_{\text{at equator}}$ ($\because R_e - R_p \approx 21$ km)

Effect of earth rotation on apparent weight:

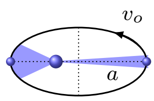
$$mg_\theta' = mg - m\omega^2 R\cos^2\theta$$



Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:



First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. ($\because \mathrm{d}\vec{L}/\mathrm{d}t = 0$).
Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

8 Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x.)

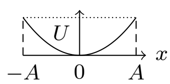
Acceleration: $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

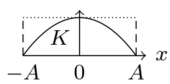
Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$

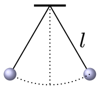


Kinetic energy $K = \frac{1}{2}mv^2$

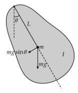


Total energy: $E = U + K = \frac{1}{2}m\omega^2A^2$

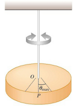
Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$



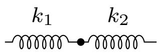
Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$



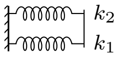
Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{k}}$



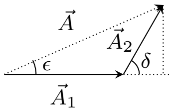
Springs in series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$



Springs in parallel: $k_{eq} = k_1 + k_2$



Superposition of two SHM's:



$$x_1 = A_1 \sin\omega t, \qquad x_2 = A_2 \sin(\omega t + \delta)$$
$$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$$
$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$
$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

9 Properties of Matter

Modulus of rigidity: $Y = \frac{F/A}{\Delta l/l}$, $B = -V\frac{\Delta P}{\Delta V}$, $\eta = \frac{F}{A\theta}$

Compressibility: $K = \frac{1}{B} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}$

Poisson's ratio: $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta l/l}$

Elastic energy: $U = \frac{1}{2}$ stress $\times$ strain $\times$ volume

Surface tension: $S = F/l$

Surface energy: $U = SA$

Excess pressure in bubble:

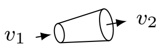
$$\Delta p_{\text{air}} = 2S/R, \quad \Delta p_{\text{soap}} = 4S/R$$

Capillary rise: $h = \frac{2S\cos\theta}{r\rho g}$

Hydrostatic pressure: $p = \rho g h$

Buoyant force: $F_B = \rho V g =$ Weight of displaced liquid

Equation of continuity: $A_1v_1 = A_2v_2$

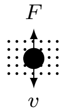


Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho g h =$ constant

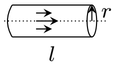
Torricelli's theorem: $v_{\text{efflux}} = \sqrt{2gh}$

Viscous force: $F = -\eta A\frac{\mathrm{d}v}{\mathrm{d}x}$

Stoke's law: $F = 6\pi\eta r v$



Poiseuilli's equation: $\frac{\text{Volume flow}}{\text{time}} = \frac{\pi p r^4}{8\eta l}$



Terminal velocity: $v_t = \frac{2r^2(\rho-\sigma)g}{9\eta}$