

Question: The benches of a gallery in a cricket stadium are 1 m wide and 1 m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit?

Solution: Consider the coordinate system shown in the figure. The ball is thrown from a height $y_0 = 1$ m with an initial velocity $u = 35$ m/s making an angle $\theta = 53^\circ$ from the horizontal. The horizontal component of the velocity $u_x = u \cos \theta \hat{i}$ remains constant. The vertical component varies due to acceleration $a_y = -g\hat{j} = -10 \text{ m/s}^2 \hat{j}$ as $v_y = (u \sin \theta - gt)\hat{j}$. The coordinate of the ball at time t are given by,

$$x = u \cos \theta t, \quad (1)$$

$$y = y_0 + u \sin \theta t - \frac{1}{2}gt^2. \quad (2)$$

Substitute t from first equation into the second equation to get,

$$y = y_0 + x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} = 1 + x \tan 53^\circ - \frac{10x^2}{2 \times 35^2 \times \cos^2 53^\circ} \quad (3)$$

$$= 1 + \frac{4}{3}x - \frac{5}{441}x^2, \quad (4)$$

where we used $\tan 53^\circ = 4/3$, $\cos 53^\circ = 3/5$, and $g = 10 \text{ m/s}^2$. Now, let us calculate the vertical height at the beginning of bench i.e., at $x_1 = 110$ m, $x_2 = 111$ m, $x_3 = 112$ m, $x_4 = 113$ m, $x_5 = 114$ m and $x_6 = 115$ m. The values we get are $y_1 = 10.48$ m, $y_2 = 9.31$ m, $y_3 = 8.11$ m, $y_4 = 6.89$ m, $y_5 = 5.65$ m and $y_6 = 4.39$ m. It is clear that ball hits the 5th bench and not the 6th as given in the answer. Note that distances shown in the figure are actual distances (scaled down by 10) i.e., you can measure each parameter by scale.

