

Question: A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of rotation

- (a) increases (b) decreases (c) remains unchanged (d) doubles

Solution: There is a little ambiguity in the question. It is not mentioned whether stool is free to rotate or not. We assume that stool is free to rotate and consider the stool and the man as a system. Let $L_{s,i}$ and $L_{m,i}$ be angular momentum of the stool and the man just before the man fold his hands. Thus, initial angular momentum of the system is $L_{s,i} + L_{m,i}$. There is no external torque on the system when the man folds his hands. Hence angular momentum of the system (stool+man) about the axis of rotation will remain constant i.e.,

$$L_{s,i} + L_{m,i} = L_{s,f} + L_{m,f}, \quad (1)$$

where $L_{s,f}$ and $L_{m,f}$ are final angular momentum of the stool and the man. Let ω_i and ω_f be initial and final angular velocities of the system (note that man firmly sitting on the stool). The moment of inertia of the stool about axis of rotation remains constant i.e., $I_{s,i} = I_{s,f} = I_s$. Initial moment of inertia of the man about axis of rotation is $I_{m,i}$ and final moment of inertia about same axis is $I_{m,f}$, where $I_{m,f} < I_{m,i}$ (the moment of inertia of the man decreases when he fold his hands). Now, $L_{s,i} = I_s\omega_i$, $L_{m,i} = I_{m,i}\omega_i$, $L_{s,f} = I_s\omega_f$, and $L_{m,f} = I_{m,f}\omega_f$. Substitute in equation 1 and simplify to get,

$$\omega_f = \frac{I_s + I_{m,i}}{I_s + I_{m,f}}\omega_i > \omega_i \quad \because (I_{m,i} < I_{m,f}). \quad (2)$$

The final angular momentum of the man can be written as

$$\begin{aligned} L_{m,f} &= I_{m,f}\omega_f = I_{m,f} \frac{I_s + I_{m,i}}{I_s + I_{m,f}}\omega_i = \frac{1 + I_s/I_{m,i}}{1 + I_s/I_{m,f}}(I_{m,i}\omega_i) \\ &= \frac{1 + I_s/I_{m,i}}{1 + I_s/I_{m,f}}(L_{m,i}) < L_{m,i} \quad (\because I_{m,i} > I_{m,f}) \end{aligned} \quad (3)$$

Thus, angular momentum of the man decrease. The answer is corrected in 2015 print of the book but it was wrong (given as remains unchanged) in older prints/editions e.g. 2011 print.