Physics formulas from Mechanics, Waves, Optics, Heat and Thermodynamics, Electricity and Magnetism and Modern Physics. Also includes the value of Physical Constants. Helps in quick revision for CBSE, NEET, JEE Mains, and Advanced.

0.1: Physical Constants

Speed of light	c	3×10^8 m/s
Planck constant	h	6.63×10^{-34} J s
	hc	1242 eV-nm
Gravitation constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.38×10^{-23} J/K
Molar gas constant	R	8.314 J/(mol K)
Avogadro's number	N_A	6.023×10^{23} mol^{-1}
Charge of electron	e	1.602×10^{-19} C
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ N/A^2
Permitivity of vacuum	ϵ_0	8.85×10^{-12} F/m
Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N m^2/C^2
Faraday constant	F	96485 C/mol
Mass of electron	m_e	9.1×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit	u	931.49 MeV/c^2
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/(m^2 K^4)
Rydberg constant	R_∞	1.097×10^7 m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J/T
Bohr radius	a_0	0.529×10^{-10} m
Standard atmosphere	atm	1.01325×10^5 Pa
Wien displacement constant	b	2.9×10^{-3} m K

1 MECHANICS

1.1: Vectors

Notation: $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:
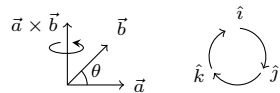
$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a} \times \vec{b}| = ab\sin\theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

$$\vec{v}_{\text{av}} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{\text{inst}} = d\vec{r}/dt$$
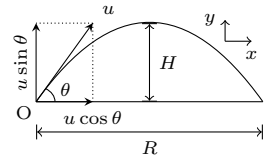$$\vec{a}_{\text{av}} = \Delta\vec{v}/\Delta t \qquad \vec{a}_{\text{inst}} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \qquad s = ut + \tfrac{1}{2}at^2, \qquad v^2 - u^2 = 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:

$$x = ut\cos\theta, \qquad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}, \quad \vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{\text{AB}} = -\vec{F}_{\text{BA}}$

Frictional force: $f_{\text{static, max}} = \mu_s N, \quad f_{\text{kinetic}} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta, \frac{v^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$
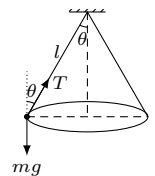
Centripetal force: $F_{\text{c}} = \frac{mv^2}{r}, \quad a_{\text{c}} = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\text{pseudo}} = -m\vec{a}_0, \quad F_{\text{centrifugal}} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\text{min, bottom}} = \sqrt{5gl}, \quad v_{\text{min, top}} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

1.4: Work, Power and Energy

Work: $W = \vec{F} \cdot \vec{S} = FS\cos\theta, \quad W = \int \vec{F} \cdot d\vec{S}$

Kinetic energy: $K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\text{gravitational}} = mgh, \quad U_{\text{spring}} = \tfrac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\text{conservative}} \cdot d\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

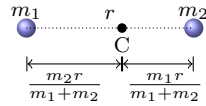**Mechanical energy:** $E = U + K$. Conserved if forces are conservative in nature.

Power $P_{av} = \frac{\Delta W}{\Delta t}, \quad P_{inst} = \vec{F} \cdot \vec{v}$
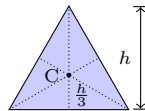
1.5: Centre of Mass and Collision

Centre of mass: $x_{cm} = \frac{\sum x_i m_i}{\sum m_i}, \quad x_{cm} = \frac{\int x \, dm}{\int dm}$
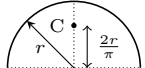
CM of few useful configurations:
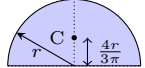
1. m_1, m_2 separated by r:

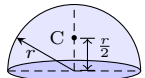2. Triangle (CM $\equiv$ Centroid) $y_c = \frac{h}{3}$
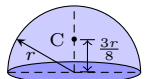
3. Semicircular ring: $y_c = \frac{2r}{\pi}$

4. Semicircular disc: $y_c = \frac{4r}{3\pi}$

5. Hemispherical shell: $y_c = \frac{r}{2}$

6. Solid Hemisphere: $y_c = \frac{3r}{8}$

7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{cm} = M\vec{v}_{cm}, \quad \vec{a}_{cm} = \frac{\vec{F}_{ext}}{M}$$

Impulse: $\vec{J} = \int \vec{F} \, dt = \Delta\vec{p}$

Collision:

Before collision   After collision
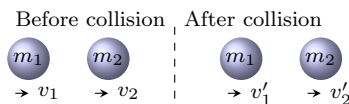
Momentum conservation: $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$
Elastic Collision: $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$
Coefficient of restitution:

$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.
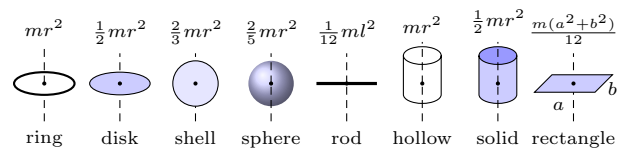
1.6: Rigid Body Dynamics

Angular velocity: $\omega_{av} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \frac{d\theta}{dt}, \quad \vec{v} = \vec{\omega} \times \vec{r}$

Angular Accel.: $\alpha_{av} = \frac{\Delta\omega}{\Delta t}, \quad \alpha = \frac{d\omega}{dt}, \quad \vec{a} = \vec{\alpha} \times \vec{r}$
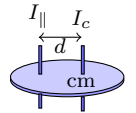
Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \frac{1}{2}\alpha t^2, \quad \omega^2 - \omega_0^2 = 2\alpha\theta$$
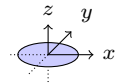
Moment of Inertia: $I = \sum_i m_i r_i^2, \quad I = \int r^2 dm$

mr^2 ring, $\frac{1}{2}mr^2$ disk, $\frac{2}{3}mr^2$ shell, $\frac{2}{5}mr^2$ sphere, $\frac{1}{12}ml^2$ rod, mr^2 hollow, $\frac{1}{2}mr^2$ solid, $\frac{m(a^2+b^2)}{12}$ rectangle

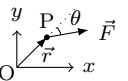**Theorem of Parallel Axes:** $I_{\parallel} = I_{cm} + md^2$

Theorem of Perp. Axes: $I_z = I_x + I_y$

Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}, \quad \vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}, \quad \vec{\tau} = \frac{d\vec{L}}{dt}, \quad \tau = I\alpha$

Conservation of $\vec{L}$: $\vec{\tau}_{ext} = 0 \implies \vec{L} = \text{const.}$

Equilibrium condition: $\sum \vec{F} = \vec{0}, \quad \sum \vec{\tau} = \vec{0}$
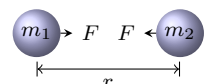
Kinetic Energy: $K_{rot} = \frac{1}{2}I\omega^2$

Dynamics:

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}, \qquad \vec{F}_{ext} = m\vec{a}_{cm}, \qquad \vec{p}_{cm} = m\vec{v}_{cm}$$
$$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2, \quad \vec{L} = I_{cm}\vec{\omega} + \vec{r}_{cm} \times m\vec{v}_{cm}$$

1.7: Gravitation

Gravitational force: $F = G\frac{m_1 m_2}{r^2}$

Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

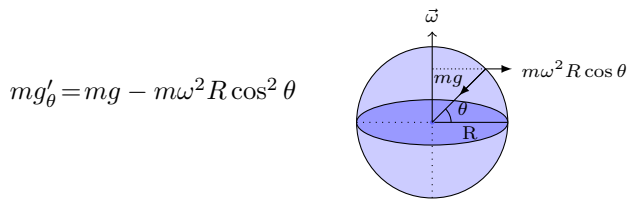**Variation of g with depth:** $g_{inside} \approx g\left(1 - \frac{2h}{R}\right)$

Variation of g with height: $g_{outside} \approx g\left(1 - \frac{h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{at\ pole} > g_{at\ equator} \quad (\because R_e - R_p \approx 21 \text{ km})$

Effect of earth rotation on apparent weight:

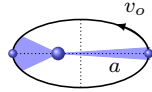$$mg'_\theta = mg - m\omega^2 R \cos^2\theta$$

Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:

> **First:** Elliptical orbit with sun at one of the focus.
> **Second:** Areal velocity is constant. ($\because \mathrm{d}\vec{L}/\mathrm{d}t = 0$).
> **Third:** $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

1.8: Simple Harmonic Motion
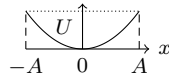
Hooke's law: $F = -kx$ (for small elongation x.)

Acceleration: $a = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$

Kinetic energy $K = \frac{1}{2}mv^2$
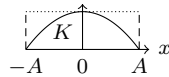
Total energy: $E = U + K = \frac{1}{2}m\omega^2 A^2$

Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$
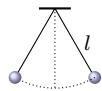
Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$

Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{k}}$
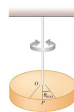
Springs in series: $\frac{1}{k_{\mathrm{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$
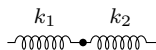
Springs in parallel: $k_{\mathrm{eq}} = k_1 + k_2$
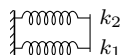
Superposition of two SHM's:
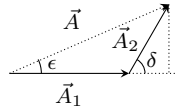
$$x_1 = A_1 \sin\omega t, \qquad x_2 = A_2\sin(\omega t + \delta)$$
$$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$$
$$A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1 A_2 \cos\delta}$$
$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

1.9: Properties of Matter

Modulus of rigidity: $Y = \frac{F/A}{\Delta l/l}$, $B = -V\frac{\Delta P}{\Delta V}$, $\eta = \frac{F}{A\theta}$

Compressibility: $K = \frac{1}{B} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}$

Poisson's ratio: $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta l/l}$

Elastic energy: $U = \frac{1}{2}\,\text{stress} \times \text{strain} \times \text{volume}$

Surface tension: $S = F/l$
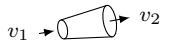
Surface energy: $U = SA$

Excess pressure in bubble:

$$\Delta p_{\mathrm{air}} = 2S/R, \quad \Delta p_{\mathrm{soap}} = 4S/R$$

Capillary rise: $h = \frac{2S\cos\theta}{r\rho g}$

Hydrostatic pressure: $p = \rho g h$

Buoyant force: $F_B = \rho V g = $ Weight of displaced liquid
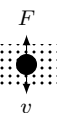
Equation of continuity: $A_1 v_1 = A_2 v_2$

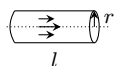**Bernoulli's equation:** $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Torricelli's theorem: $v_{\mathrm{efflux}} = \sqrt{2gh}$

Viscous force: $F = -\eta A\frac{\mathrm{d}v}{\mathrm{d}x}$

Stoke's law: $F = 6\pi\eta r v$

Poiseuilli's equation: $\frac{\text{Volume flow}}{\text{time}} = \frac{\pi p r^4}{8\eta l}$

Terminal velocity: $v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$
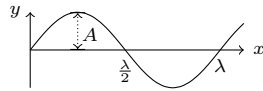
2 Waves

2.1: Waves Motion

General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

Notation: Amplitude A, Frequency ν, Wavelength λ, Period T, Angular Frequency ω, Wave Number k,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v:

$$y = f(t - x/v), \rightsquigarrow +x; \quad y = f(t + x/v), \rightsquigarrow -x$$

Progressive sine wave:

$$y = A\sin(kx - \omega t) = A\sin(2\pi(x/\lambda - t/T))$$

2.2: Waves on a String

Speed of waves on a string with mass per unit length μ and tension T: $v = \sqrt{T/\mu}$

Transmitted power: $P_{\text{av}} = 2\pi^2 \mu v A^2 \nu^2$

Interference:

$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx - \omega t + \delta)$$

$$\boxed{y = y_1 + y_2 = A\sin(kx - \omega t + \epsilon)}$$

$$A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1 A_2\cos\delta}$$

$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:
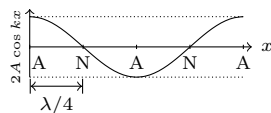
$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx + \omega t)$$

$$\boxed{y = y_1 + y_2 = (2A\cos kx)\sin\omega t}$$
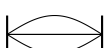
$$x = \begin{cases} \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, & \text{nodes;} \quad n = 0, 1, 2, \dots \\ n\frac{\lambda}{2}, & \text{antinodes.} \quad n = 0, 1, 2, \dots \end{cases}$$

String fixed at both ends:
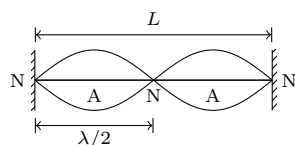
1. Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$
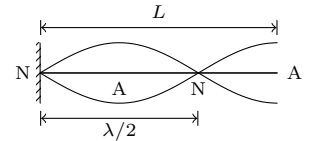2. Allowed Freq.: $L = n\frac{\lambda}{2}$, $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \dots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$
4. 1st overtone/2nd harmonics: $\nu_1 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$
5. 2nd overtone/3rd harmonics: $\nu_2 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$
6. All harmonics are present.

String fixed at one end:

1. Boundary conditions: $y = 0$ at $x = 0$
2. Allowed Freq.: $L = (2n + 1)\frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L}\sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \dots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$
4. 1st overtone/3rd harmonics: $\nu_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$
5. 2nd overtone/5th harmonics: $\nu_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$
6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$. $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

2.3: Sound Waves

Displacement wave: $s = s_0\sin\omega(t - x/v)$

Pressure wave: $p = p_0\cos\omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$
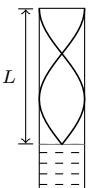
Intensity: $I = \frac{2\pi^2 B}{v}s_0{}^2\nu^2 = \frac{p_0{}^2 v}{2B} = \frac{p_0{}^2}{2\rho v}$

Standing longitudinal waves:

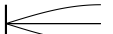$$p_1 = p_0\sin\omega(t - x/v), \quad p_2 = p_0\sin\omega(t + x/v)$$
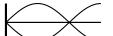$$p = p_1 + p_2 = 2p_0\cos kx\sin\omega t$$

Closed organ pipe:

1. Boundary condition: $y = 0$ at $x = 0$
2. Allowed freq.: $L = (2n + 1)\frac{\lambda}{4}$, $\nu = (2n + 1)\frac{v}{4L}$, $n = 0, 1, 2, \dots$
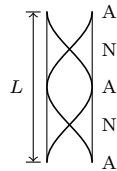3. Fundamental/1st harmonics: $\nu_0 = \frac{v}{4L}$
4. 1st overtone/3rd harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$

5. 2nd overtone/5th harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$

6. Only odd harmonics are present.

Open organ pipe:

1. Boundary condition: $y = 0$ at $x = 0$
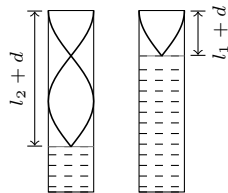   Allowed freq.: $L = n\frac{\lambda}{2}$, $\nu = n\frac{v}{4L}$, $n = 1, 2, \ldots$

2. Fundamental/1st harmonics: $\nu_0 = \frac{v}{2L}$

3. 1st overtone/2nd harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$

4. 2nd overtone/3rd harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$

5. All harmonics are present.

Resonance column:

$$l_1 + d = \frac{\lambda}{2}, \quad l_2 + d = \frac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

$$p_1 = p_0 \sin\omega_1(t - x/v), \quad p_2 = p_0 \sin\omega_2(t - x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos\Delta\omega(t - x/v)\sin\omega(t - x/v)$$
$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad \text{(beats freq.)}$$

Doppler Effect:

$$\nu = \frac{v + u_o}{v - u_s}\nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

2.4: Light Waves

Plane Wave: $E = E_0 \sin\omega(t - \frac{x}{v})$, $I = I_0$

Spherical Wave: $E = \frac{aE_0}{r}\sin\omega(t - \frac{r}{v})$, $I = \frac{I_0}{r^2}$

Young's double slit experiment

Path difference: $\Delta x = \frac{dy}{D}$
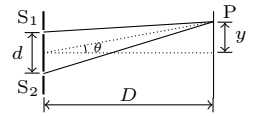
Phase difference: $\delta = \frac{2\pi}{\lambda}\Delta x$

Interference Conditions: for integer n,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta,$$
$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2, \; I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_1 = I_2 : I = 4I_0 \cos^2\frac{\delta}{2}, \; I_{\max} = 4I_0, \; I_{\min} = 0$$
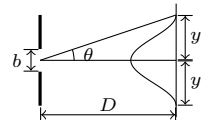
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu\Delta x$

Interference of waves transmitted through thin film:

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive.} \end{cases}$$
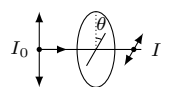
Diffraction from a single slit:

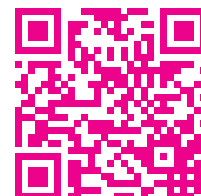For Minima: $n\lambda = b\sin\theta \approx b(y/D)$

Resolution: $\sin\theta = \frac{1.22\lambda}{b}$

Law of Malus: $I = I_0 \cos^2\theta$

3   Optics

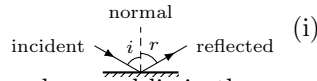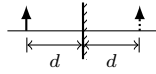## 3.1: Reflection of Light
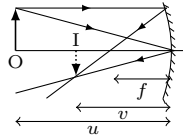
Laws of reflection:

(i) Incident ray, reflected ray, and normal lie in the same plane (ii) $\angle i = \angle r$

Plane mirror:

(i) the image and the object are equidistant from mirror (ii) virtual image of real object
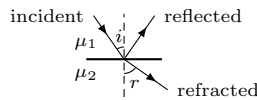
Spherical Mirror:

1. Focal length $f = R/2$
2. Mirror equation: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
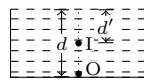3. Magnification: $m = -\frac{v}{u}$

3.2: Refraction of Light

Refractive index: $\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$

Snell's Law: $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

Apparent depth: $\mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{d}{d'}$

Critical angle: $\theta_c = \sin^{-1} \frac{1}{\mu}$
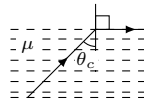
Deviation by a prism:
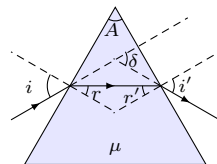
$$\delta = i + i' - A, \quad \text{general result}$$
$$\mu = \frac{\sin \frac{A+\delta_m}{2}}{\sin \frac{A}{2}}, \quad i = i' \text{ for minimum deviation}$$

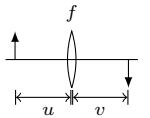$$\delta_m = (\mu - 1)A, \quad \text{for small } A$$
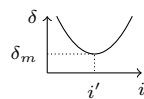
Refraction at spherical surface:
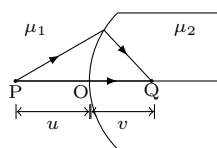
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}, \quad m = \frac{\mu_1 v}{\mu_2 u}$$

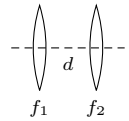**Lens maker's formula:** $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Lens formula: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad m = \frac{v}{u}$

Power of the lens: $P = \frac{1}{f}$, P in diopter if f in metre.

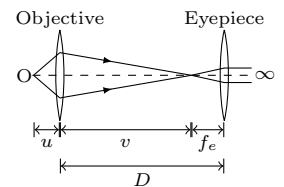**Two thin lenses separated by distance d:**

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$
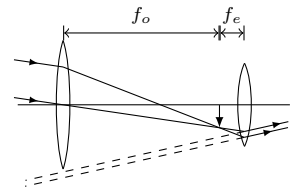
3.3: Optical Instruments

Simple microscope: $m = D/f$ in normal adjustment.

Compound microscope:

1. Magnification in normal adjustment: $m = \frac{v}{u} \frac{D}{f_e}$
2. Resolving power: $R = \frac{1}{\Delta d} = \frac{2\mu \sin \theta}{\lambda}$

Astronomical telescope:

1. In normal adjustment: $m = -\frac{f_o}{f_e}$, $L = f_o + f_e$
2. Resolving power: $R = \frac{1}{\Delta \theta} = \frac{1}{1.22\lambda}$

3.4: Dispersion

Cauchy's equation: $\mu = \mu_0 + \frac{A}{\lambda^2}, \quad A > 0$
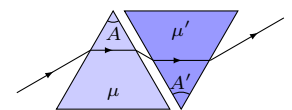
Dispersion by prism with small A and i:

1. Mean deviation: $\delta_y = (\mu_y - 1)A$
2. Angular dispersion: $\theta = (\mu_v - \mu_r)A$

Dispersive power: $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \approx \frac{\theta}{\delta_y}$ (if A and i small)

Dispersion without deviation:

$$(\mu_y - 1)A + (\mu_y' - 1)A' = 0$$

Deviation without dispersion:
$$(\mu_v - \mu_r)A = (\mu_v' - \mu_r')A'$$

4　Heat and Thermodynamics

4.1: Heat and Temperature

Temp. scales: $F = 32 + \frac{9}{5}C, \quad K = C + 273.16$

Ideal gas equation: $pV = nRT, \quad n:$ number of moles

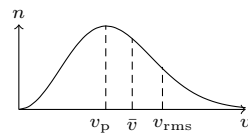**van der Waals equation:** $\left(p + \frac{a}{V^2}\right)(V - b) = nRT$

Thermal expansion: $L = L_0(1 + \alpha\Delta T),$
$A = A_0(1 + \beta\Delta T), V = V_0(1 + \gamma\Delta T), \gamma = 2\beta = 3\alpha$

Thermal stress of a material: $\frac{F}{A} = Y\frac{\Delta l}{l}$

4.2: Kinetic Theory of Gases

General: $M = mN_A, k = R/N_A$

Maxwell distribution of speed:

RMS speed: $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$

Average speed: $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$

Most probable speed: $v_p = \sqrt{\frac{2kT}{m}}$

Pressure: $p = \frac{1}{3}\rho v_{rms}^2$

Equipartition of energy: $K = \frac{1}{2}kT$ for each degree of freedom. Thus, $K = \frac{f}{2}kT$ for molecule having f degrees of freedoms.

Internal energy of n moles of an ideal gas is $U = \frac{f}{2}nRT$.

4.3: Specific Heat

Specific heat: $s = \frac{Q}{m\Delta T}$

Latent heat: $L = Q/m$

Specific heat at constant volume: $C_v = \left.\frac{\Delta Q}{n\Delta T}\right|_V$

Specific heat at constant pressure: $C_p = \left.\frac{\Delta Q}{n\Delta T}\right|_p$

Relation between C_p and C_v: $C_p - C_v = R$

Ratio of specific heats: $\gamma = C_p/C_v$

Relation between U and C_v: $\Delta U = nC_v\Delta T$

Specific heat of gas mixture:

$$C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}, \quad \gamma = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$$

Molar internal energy of an ideal gas: $U = \frac{f}{2}RT$, $f = 3$ for monatomic and $f = 5$ for diatomic gas.

4.4: Theromodynamic Processes

First law of thermodynamics: $\Delta Q = \Delta U + \Delta W$

Work done by the gas:

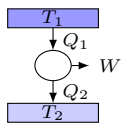$$\Delta W = p\Delta V, \quad W = \int_{V_1}^{V_2} p\,\mathrm{d}V$$

$$W_{\text{isothermal}} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$W_{\text{isobaric}} = p(V_2 - V_1)$$

$$W_{\text{adiabatic}} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

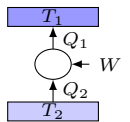$$W_{\text{isochoric}} = 0$$

Efficiency of the heat engine:

$$\eta = \frac{\text{work done by the engine}}{\text{heat supplied to it}} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{\text{carnot}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Coeff. of performance of refrigerator:

$$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

Entropy: $\Delta S = \frac{\Delta Q}{T}, S_f - S_i = \int_i^f \frac{\Delta Q}{T}$

Const. $T : \Delta S = \frac{Q}{T}, \quad$ Varying $T : \Delta S = ms\ln\frac{T_f}{T_i}$
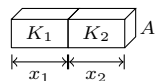
Adiabatic process: $\Delta Q = 0, pV^\gamma = $ constant
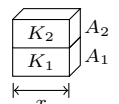
4.5: Heat Transfer

Conduction: $\frac{\Delta Q}{\Delta t} = -KA\frac{\Delta T}{x}$

Thermal resistance: $R = \frac{x}{KA}$

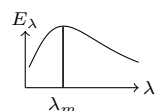$$R_{\text{series}} = R_1 + R_2 = \frac{1}{A}\left(\frac{x_1}{K_1} + \frac{x_2}{K_2}\right)$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{x}(K_1 A_1 + K_2 A_2)$$

Kirchhoff's Law: $\frac{\text{emissive power}}{\text{absorptive power}} = \frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{blackbody}}$

Wien's displacement law: $\lambda_m T = b$

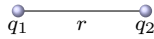**Stefan-Boltzmann law:** $\frac{\Delta Q}{\Delta t} = \sigma e A T^4$

Newton's law of cooling: $\frac{\mathrm{d}T}{\mathrm{d}t} = -bA(T - T_0)$
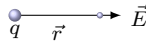
5 Electricity and Magnetism

5.1: Electrostatics

Coulomb's law: $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$

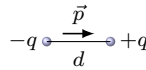**Electric field:** $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

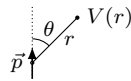**Electrostatic energy:** $U = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

Electrostatic potential: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$$dV = -\vec{E} \cdot \vec{r}, \quad V(\vec{r}) = -\int_\infty^{\vec{r}} \vec{E} \cdot d\vec{r}$$

Electric dipole moment: $\vec{p} = q\vec{d}$

Potential of a dipole: $V = \frac{1}{4\pi\epsilon_0} \frac{p\cos\theta}{r^2}$

Field of a dipole:

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{2p\cos\theta}{r^3}, \quad E_\theta = \frac{1}{4\pi\epsilon_0} \frac{p\sin\theta}{r^3}$$
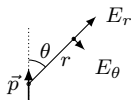
Torque on a dipole placed in $\vec{E}$: $\vec{\tau} = \vec{p} \times \vec{E}$

Pot. energy of a dipole placed in $\vec{E}$: $U = -\vec{p} \cdot \vec{E}$

5.2: Gauss's Law and its Applications

Electric flux: $\phi = \oint \vec{E} \cdot d\vec{S}$

Gauss's law: $\oint \vec{E} \cdot d\vec{S} = q_{in}/\epsilon_0$

Field of a uniformly charged ring on its axis:

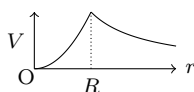$$E_P = \frac{1}{4\pi\epsilon_0} \frac{qx}{(a^2+x^2)^{3/2}}$$
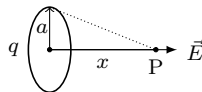
E and V of a uniformly charged sphere:
$$E = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$
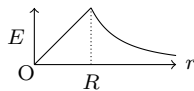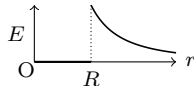
$$V = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Qr^2}{R^3}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r}, & \text{for } r \geq R \end{cases}$$
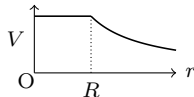
E and V of a uniformly charged spherical shell:
$$E = \begin{cases} 0, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$

$$V = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{R}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0} \frac{Q}{r}, & \text{for } r \geq R \end{cases}$$

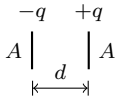**Field of a line charge:** $E = \frac{\lambda}{2\pi\epsilon_0 r}$

Field of an infinite sheet: $E = \frac{\sigma}{2\epsilon_0}$

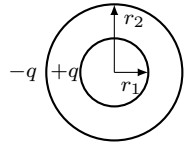**Field in the vicinity of conducting surface:** $E = \frac{\sigma}{\epsilon_0}$

5.3: Capacitors
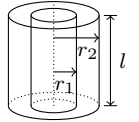
Capacitance: $C = q/V$

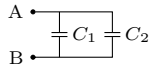**Parallel plate capacitor:** $C = \epsilon_0 A/d$

Spherical capacitor: $C = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$
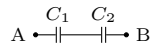
Cylindrical capacitor: $C = \frac{2\pi\epsilon_0 l}{\ln(r_2/r_1)}$

Capacitors in parallel: $C_{eq} = C_1 + C_2$

Capacitors in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Force between plates of a parallel plate capacitor:
$$F = \frac{Q^2}{2A\epsilon_0}$$

Energy stored in capacitor: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy density in electric field E: $U/V = \frac{1}{2}\epsilon_0 E^2$

Capacitor with dielectric: $C = \frac{\epsilon_0 K A}{d}$

5.4: Current electricity

Current density: $j = i/A = \sigma E$

Drift speed: $v_d = \frac{1}{2}\frac{eE}{m}\tau = \frac{i}{neA}$
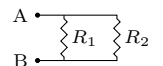
Resistance of a wire: $R = \rho l/A$, where $\rho = 1/\sigma$

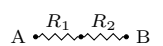**Temp. dependence of resistance:** $R = R_0(1 + \alpha\Delta T)$

Ohm's law: $V = iR$

Kirchhoff's Laws: *(i) The Junction Law:* The algebraic sum of all the currents directed towards a node is zero i.e., $\Sigma_{node} I_i = 0$. *(ii) The Loop Law:* The algebraic sum of all the potential differences along a closed loop in a circuit is zero i.e., $\Sigma_{loop}\Delta V_i = 0$.
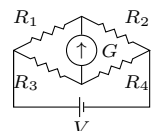
Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Resistors in series: $R_{eq} = R_1 + R_2$
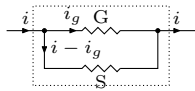
Wheatstone bridge:

Balanced if $R_1/R_2 = R_3/R_4$.

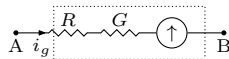**Electric Power:** $P = V^2/R = I^2 R = IV$
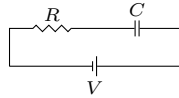
Galvanometer as an Ammeter:

$$i_g G = (i - i_g)S$$

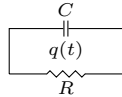**Galvanometer as a Voltmeter:**

$$V_{AB} = i_g(R + G)$$

Charging of capacitors:

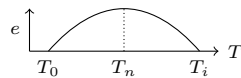$$q(t) = CV\left[1 - e^{-\frac{t}{RC}}\right]$$

Discharging of capacitors: $q(t) = q_0 e^{-\frac{t}{RC}}$

Time constant in RC circuit: $\tau = RC$

Peltier effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{charge transferred}}$.

Seeback effect:

1. Thermo-emf: $e = aT + \frac{1}{2}bT^2$
2. Thermoelectric power: $de/dt = a + bT$.
3. Neutral temp.: $T_n = -a/b$.
4. Inversion temp.: $T_i = -2a/b$.

Thomson effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Thomson heat}}{\text{charge transferred}} = \sigma \Delta T$.

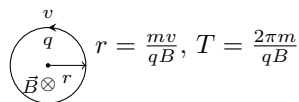**Faraday's law of electrolysis:** The mass deposited is

$$m = Zit = \frac{1}{F}Eit$$

where i is current, t is time, Z is electrochemical equivalent, E is chemical equivalent, and $F = 96485$ C/g is Faraday constant.
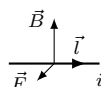
5.5: Magnetism

Lorentz force on a moving charge: $\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$
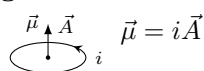
Charged particle in a uniform magnetic field:

$$r = \frac{mv}{qB}, \quad T = \frac{2\pi m}{qB}$$

Force on a current carrying wire:

$$\vec{F} = i\,\vec{l} \times \vec{B}$$

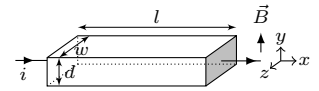**Magnetic moment of a current loop (dipole):**

$$\vec{\mu} = i\vec{A}$$

Torque on a magnetic dipole placed in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$
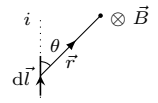
Energy of a magnetic dipole placed in $\vec{B}$:

$$U = -\vec{\mu} \cdot \vec{B}$$
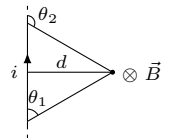
Hall effect: $V_w = \frac{Bi}{ned}$

5.6: Magnetic Field due to Current

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \vec{r}}{r^3}$

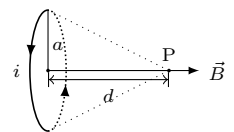**Field due to a straight conductor:**

$$B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 - \cos\theta_2)$$

Field due to an infinite straight wire: $B = \frac{\mu_0 i}{2\pi d}$

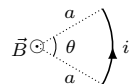**Force between parallel wires:** $\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}$

Field on the axis of a ring:

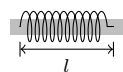$$B_P = \frac{\mu_0 i a^2}{2(a^2 + d^2)^{3/2}}$$

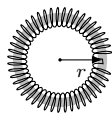**Field at the centre of an arc:** $B = \frac{\mu_0 i \theta}{4\pi a}$

Field at the centre of a ring: $B = \frac{\mu_0 i}{2a}$

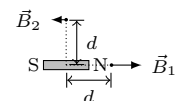**Ampere's law:** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$

Field inside a solenoid: $B = \mu_0 n i$, $n = \frac{N}{l}$

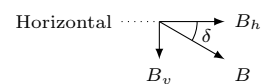**Field inside a toroid:** $B = \frac{\mu_0 N i}{2\pi r}$

Field of a bar magnet:

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}, \quad B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Angle of dip: $B_h = B\cos\delta$

Tangent galvanometer: $B_h \tan\theta = \frac{\mu_0 n i}{2r}$, $\quad i = K\tan\theta$

Moving coil galvanometer: $niAB = k\theta$, $\quad i = \frac{k}{nAB}\theta$

Time period of magnetometer: $T = 2\pi\sqrt{\frac{I}{MB_h}}$

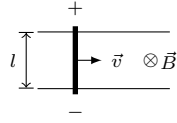**Permeability:** $\vec{B} = \mu\vec{H}$

5.7: Electromagnetic Induction

Magnetic flux: $\phi = \oint \vec{B} \cdot \mathrm{d}\vec{S}$

Faraday's law: $e = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$

Lenz's Law: Induced current create a B-field that opposes the change in magnetic flux.

Motional emf: $e = Blv$

Self inductance: $\phi = Li$, $e = -L\frac{\mathrm{d}i}{\mathrm{d}t}$

Self inductance of a solenoid: $L = \mu_0 n^2 (\pi r^2 l)$

Growth of current in LR circuit: $i = \frac{e}{R}\left[1 - e^{-\frac{t}{L/R}}\right]$
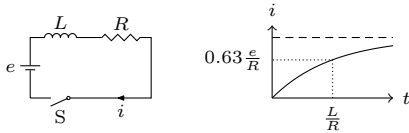
Decay of current in LR circuit: $i = i_0 e^{-\frac{t}{L/R}}$
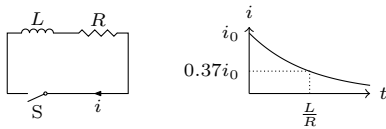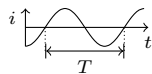

Time constant of LR circuit: $\tau = L/R$

Energy stored in an inductor: $U = \frac{1}{2}Li^2$

Energy density of B field: $u = \frac{U}{V} = \frac{B^2}{2\mu_0}$

Mutual inductance: $\phi = Mi$, $e = -M\frac{\mathrm{d}i}{\mathrm{d}t}$

EMF induced in a rotating coil: $e = NAB\omega \sin \omega t$

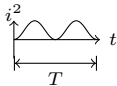**Alternating current:**

$i = i_0 \sin(\omega t + \phi)$, $T = 2\pi/\omega$

Average current in AC: $\bar{i} = \frac{1}{T}\int_0^T i \, \mathrm{d}t = 0$

RMS current: $i_{\mathrm{rms}} = \left[\frac{1}{T}\int_0^T i^2 \, \mathrm{d}t\right]^{1/2} = \frac{i_0}{\sqrt{2}}$
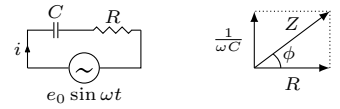

Energy: $E = i_{\mathrm{rms}}^2 RT$

Capacitive reactance: $X_c = \frac{1}{\omega C}$

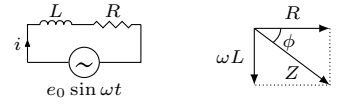**Inductive reactance:** $X_L = \omega L$
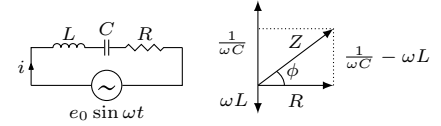
Imepedance: $Z = e_0/i_0$

RC circuit:

$Z = \sqrt{R^2 + (1/\omega C)^2}$, $\tan \phi = \frac{1}{\omega CR}$

LR circuit:

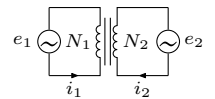$Z = \sqrt{R^2 + \omega^2 L^2}$, $\tan \phi = \frac{\omega L}{R}$

LCR Circuit:

$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$, $\tan \phi = \frac{\frac{1}{\omega C} - \omega L}{R}$

$\nu_{\mathrm{resonance}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

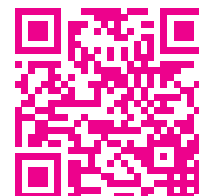**Power factor:** $P = e_{rms} i_{rms} \cos \phi$

Transformer: $\frac{N_1}{N_2} = \frac{e_1}{e_2}$, $e_1 i_1 = e_2 i_2$

Speed of the EM waves in vacuum: $c = 1/\sqrt{\mu_0 \epsilon_0}$

6   Modern Physics
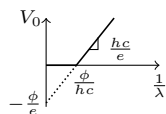
6.1: Photo-electric effect

Photon's energy: $E = h\nu = hc/\lambda$

Photon's momentum: $p = h/\lambda = E/c$

Max. KE of ejected photo-electron: $K_{\max} = h\nu - \phi$

Threshold freq. in photo-electric effect: $\nu_0 = \phi/h$

Stopping potential: $V_o = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$

de Broglie wavelength: $\lambda = h/p$

6.2: The Atom

Energy in nth Bohr's orbit:

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0{}^2h^2n^2}, \quad E_n = -\frac{13.6Z^2}{n^2}\text{ eV}$$

Radius of the nth Bohr's orbit:

$$r_n = \frac{\epsilon_0 h^2 n^2}{\pi m Z e^2}, \quad r_n = \frac{n^2 a_0}{Z}, \quad a_0 = 0.529\text{ Å}$$

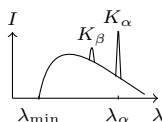**Quantization of the angular momentum:** $l = \frac{nh}{2\pi}$

Photon energy in state transition: $E_2 - E_1 = h\nu$

Wavelength of emitted radiation: for a transition from nth to mth state:

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

X-ray spectrum: $\lambda_{\min} = \frac{hc}{eV}$

Moseley's law: $\sqrt{\nu} = a(Z - b)$

X-ray diffraction: $2d\sin\theta = n\lambda$
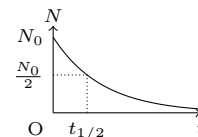
Heisenberg uncertainity principle:
$\Delta p \Delta x \geq h/(2\pi), \qquad \Delta E \Delta t \geq h/(2\pi)$

6.3: The Nucleus

Nuclear radius: $R = R_0 A^{1/3}, \quad R_0 \approx 1.1 \times 10^{-15}$ m

Decay rate: $\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$

Population at time t: $N = N_0 e^{-\lambda t}$

Half life: $t_{1/2} = 0.693/\lambda$

Average life: $t_{\mathrm{av}} = 1/\lambda$

Population after n half lives: $N = N_0/2^n$.

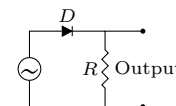**Mass defect:** $\Delta m = [Zm_p + (A - Z)m_n] - M$

Binding energy: $B = [Zm_p + (A - Z)m_n - M]c^2$
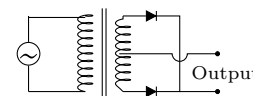
Q-value: $Q = U_i - U_f$

Energy released in nuclear reaction: $\Delta E = \Delta mc^2$ where $\Delta m = m_{\mathrm{reactants}} - m_{\mathrm{products}}$.

6.4: Vacuum tubes and Semiconductors

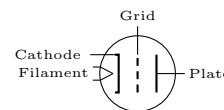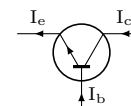**Half Wave Rectifier:**

Full Wave Rectifier:

Triode Valve:

Plate resistance of a triode: $r_p = \frac{\Delta V_p}{\Delta i_p}\Big|_{\Delta V_g = 0}$

Transconductance of a triode: $g_m = \frac{\Delta i_p}{\Delta V_g}\Big|_{\Delta V_p = 0}$

Amplification by a triode: $\mu = -\frac{\Delta V_p}{\Delta V_g}\Big|_{\Delta i_p = 0}$

Relation between r_p, μ, and g_m: $\mu = r_p \times g_m$

Current in a transistor: $I_e = I_b + I_c$

α and β parameters of a transistor: $\alpha = \frac{I_c}{I_e}$, $\beta = \frac{I_c}{I_b}$, $\beta = \frac{\alpha}{1-\alpha}$

Transconductance: $g_m = \frac{\Delta I_c}{\Delta V_{be}}$

Logic Gates:

		AND	OR	NAND	NOR	XOR
A	B	AB	A+B	$\overline{AB}$	$\overline{A+B}$	$A\bar{B} + \bar{A}B$
0	0	0	0	1	1	0
0	1	0	1	1	0	1
1	0	0	1	1	0	1
1	1	1	1	0	0	0