IIT JEE (Advanced) 2016 Paper 2: PHYSICS

SECTION 1 (Maximum Marks: 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:0	If none of the bubbles is darkened.
Negative Marks	:-1	In all other cases.

Question 1. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}.$$

The measured masses of the neutron, ${}^{1}_{1}\mathrm{H}$, ${}^{15}_{7}\mathrm{N}$, ${}^{15}_{8}\mathrm{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2/(4\pi\epsilon_0) = 1.44$ MeV-fm. Assuming that the difference between the binding energies of ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is [1 fm = 10^{-15} m] (2016)

(A) 2.85 fm (B) 3.03 fm (C) 3.42 fm (D) 3.80 fm

Solution. The binding energies of ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ are given by

$$\begin{aligned}
\mathrm{BE}_N &= \Delta m_N c^2 = (8m_n + 7m_p - M_N)c^2 \\
&= (8\times 1.008665 + 7\times 1.007825 - 15.000109)\times 931.5 \\
&= 115.49\ \mathrm{MeV}, \\
\mathrm{BE}_O &= \Delta m_O c^2 = (7m_n + 8m_p - M_O)c^2 \\
&= (7\times 1.008665 + 8\times 1.007825 - 15.003065)\times 931.5 \\
&= 111.95\ \mathrm{MeV}.
\end{aligned}$$

The difference in binding energies of ${}^{15}_{7}\mathrm{N}$ and ${}^{15}_{8}\mathrm{O}$ is

$$\Delta\mathrm{BE} = \mathrm{BE}_N - \mathrm{BE}_O = 115.49 - 111.95 = 3.54\ \mathrm{MeV}. \quad (1)$$

The electrostatic energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ are given by

$$E_N = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R} = \frac{3}{5}\,\frac{7(7-1)1.44}{R} = \frac{36.288}{R}\ \text{MeV} \qquad (2)$$

text–fm

$$E_O = \frac{3}{5}\,\frac{8(8-1)1.44}{R} = \frac{48.384}{R}\ \text{MeV-fm.}$$

The difference in electrostatic energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ is

$$\Delta E = E_O - E_N = (12.096/R)\ \text{MeV-fm.} \qquad (3)$$

Since $\Delta E = \Delta\text{BE}$, equations (1) and (3) give $R = 12.096/3.54 = 3.42$ fm.

Question 2. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? (2016)
(A) 64 (B) 90 (C) 108 (D) 120

Solution. The activity of a radioactive sample at a time t is given by $A = A_0 e^{-\lambda t}$, where A_0 is the initial activity and λ is the decay rate. The decay rate is related to the half life of the sample by $\lambda = 0.693/t_{1/2} = (\ln 2)/t_{1/2}$. Let A_1 be the activity measured in the laboratory test at time t_1. Let the laboratory becomes safe at time t_2 i.e., activity at time t_2 is $A_2 = A_1/64$. Apply the formula for activity to get

$$A_1 = A_0 e^{-\lambda t_1}, \qquad\qquad A_2 = A_0 e^{-\lambda t_2}.$$

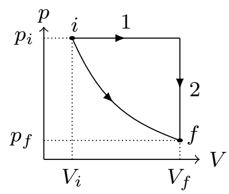
Divide A_1 by A_2 to get

$$A_1/A_2 = 64 = e^{\lambda(t_2 - t_1)}. \qquad (1)$$

Take logarithm on both sides of equation (1) and substitute $\lambda = (\ln 2)/t_{1/2} = (\ln 2)/18$ to get $t_2 - t_1 = 6(18) = 108$ days. Thus, the laboratory can be considered safe after 108 days of the test.

Question 3. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $p_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state $p_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that $p^3V^5 =$ constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at p_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately (2016)
(A) 112 J (B) 294 J (C) 588 J (D) 813 J

Solution. The equation, p^3V^5 = constant, of given adiabatic quasi-static process can be written as $pV^{5/3}$ = constant. Compare this with equation for adiabatic process, pV^γ = constant, to get ratio of the specific heats $\gamma = 5/3$. Let f be the degree of freedom of the molecules of the gas. The internal energy of one mole gas at temperature T is given by $U = \frac{f}{2}RT$. The specific heats at constant volume C_v, specific heat at constant pressure C_p, and the ratio of the specific heats are given by



$$C_v = \frac{\mathrm{d}U}{\mathrm{d}T} = \frac{f}{2}R, \quad C_p = C_v + R = \frac{f+2}{2}R, \quad \gamma = \frac{C_p}{C_v} = \frac{f+2}{f}.$$

Substitute $\gamma = 5/3$ to get $f = 3$ (given gas is monatomic). The increase in internal energy of n moles of the gas when it goes from the initial state (p_i, V_i, T_i) to the final state (p_f, V_f, T_f) is given by

$$\begin{aligned}\Delta U &= U_f - U_i = \tfrac{f}{2}(nRT_f - nRT_i) = \tfrac{f}{2}\left(p_fV_f - p_iV_i\right)\\ &= \tfrac{3}{2}\left(\tfrac{1}{32} \times 10^5 \times 8 \times 10^{-3} - 10^5 \times 10^{-3}\right) = -112.5 \text{ J}. \qquad (1)\end{aligned}$$

Negative sign of ΔU indicates that the internal energy of final state is less than that of the initial state. The work done *by* the gas in the isochoric process is zero and in the isobaric process is

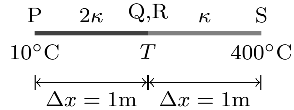
$$W = p_i(V_f - V_i) = 10^5(8 \times 10^{-3} - 10^{-3}) = 700 \text{ J}. \qquad (2)$$

The change in internal energy ΔU is same in the two processes because it is a state function. Apply first law of thermodynamics, $\Delta Q = \Delta U + W$, to get heat supplied to the gas as

$$\Delta Q = \Delta U + \Delta W = -112.5 + 700 = 587.5 \text{ J}.$$

Question 4. The end Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10 °C. Now the end P is maintained at 10 °C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is (2016)
(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

Solution. Let steady state temperature of the junction (Q,R) be T. In steady state, the rate of heat flow from the end S to R is equal to the rate of heat flow from the end Q to P because the system is insulated from the surrounding.

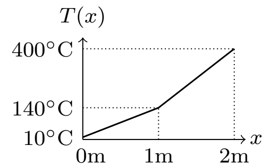


Thus,

$$\frac{\mathrm{d}Q_{SR}}{\mathrm{d}t} = \frac{\mathrm{d}Q_{QP}}{\mathrm{d}t}, \qquad \text{i.e.,} \qquad \frac{\kappa A(400 - T)}{\Delta x} = \frac{2\kappa A(T - 10)}{\Delta x},$$

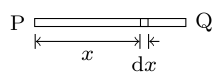
which gives $T = 140\ ^\circ\mathrm{C}$.

Let P be the origin and x-axis points towards Q. In steady state, the temperature on the wire varies linearly with the distance x i.e., $T(x) = mx + c$, where m is the slope and c is the intercept of T axis. In wire PQ, the temperature $T = 10\ ^\circ\mathrm{C}$ at $x = 0$ m and the temperature $T = 140\ ^\circ\mathrm{C}$ at $x = 1$ m. Substitute these values in $T(x) = mx + c$ to get $c = 10\ ^\circ\mathrm{C}$ and $m = 130\ ^\circ\mathrm{C/m}$. Thus, steady state temperature on the wire PQ is given by



$$T(x) = 130x + 10.$$

Now, consider a small element $\mathrm{d}x$ of the wire PQ located at a distance x from the end P. Initial temperature of this element was $10\ ^\circ\mathrm{C}$ and final temperature (in steady state) is $T(x) = 130x + 10$. The coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\ \mathrm{K}^{-1}$. The increase in length of this element due to thermal expansion is given by



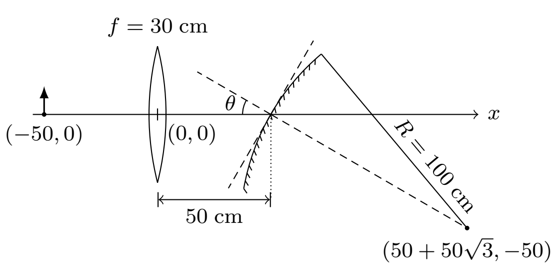
$$\mathrm{d}l = \alpha \Delta T \mathrm{d}x = \alpha(130x + 10 - 10)\mathrm{d}x = 130\alpha x \mathrm{d}x$$

Integrate from $x = 0$ m to $x = 1$ m to get increase in length as

$$l = \int_0^1 130\alpha x \mathrm{d}x = 130\alpha \left[\frac{x^2}{2}\right]_0^1 = 130(1.2 \times 10^{-5})\frac{1}{2} = 0.78\ \mathrm{mm}.$$

Detailed analysis of heat transfer reveals that temperature obeys the differential equation $\frac{1}{c^2}\frac{\partial T}{\partial t} = \frac{\mathrm{d}^2 T}{\mathrm{d}x^2}$, where c is a constant. In steady state, temperature is independent of time i.e., $\frac{\partial T}{\partial t} = 0$ which gives $\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = 0$. Integrate it twice to get $T(x) = mx + c$.

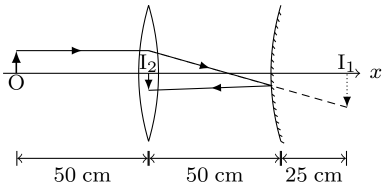
Question 5. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.



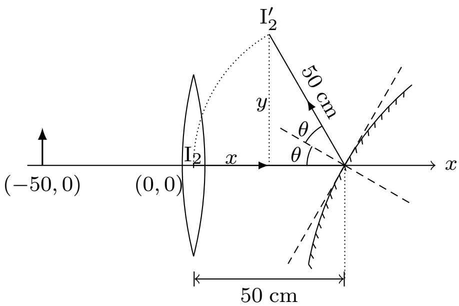
If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are (2016)

(A) $(0, 0)$ (B) $(50 - 25\sqrt{3}, 25)$ (C) $(25, 25\sqrt{3})$ (D) $(125/3, 25/\sqrt{3})$

Solution. The object distance for the lens of focal length $f = 30$ cm is $u = -50$ cm. Apply lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, to get the image distance $v = 75$ cm. Thus, image I_1 by the lens is formed 25 cm behind the mirror.



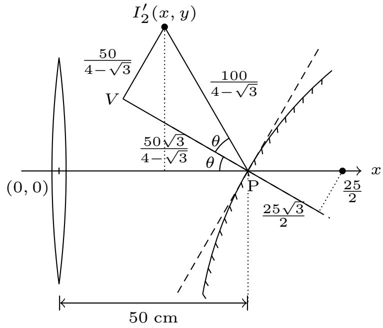
Let us consider the case when mirror is not tilted. The image I_1 will act as a virtual object with $u = 25$ cm for a convex mirror of focal length $f_m = R/2 = 50$ cm. Apply the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f_m}$, to get the image distance $v = -50$ cm. Thus, image I_2 by the mirror is formed 50 cm left of the mirror (see figure).



What happens to the image if mirror is tilted by an angle θ? Consider a ray along the principal axis of the lens. If mirror is not tilted then this ray starts from the object, refracts through the lens without deviation, incident normally on the mirror and finally comes to the image I_2 after reflection from the mirror. If the mirror is tilted by an angle θ then this ray is incident on the mirror at an angle of incidence θ and reflected by the mirror with an angle of reflection θ and finally forms a new image at I_2' (see figure). Thus, the axis on which image lies makes an angle $2\theta = 60^\circ$ with the principal axis of the lens. Also, the image distance will remain same upto the first order of approximation (image distance will remain exactly same in case of plane mirror). Thus, (x, y) coordinates of the image are

$$x = 50 - 50\cos 60^\circ = 25 \text{ cm}, \qquad y = 50 \sin 60^\circ = 25\sqrt{3} \text{ cm}.$$

Aliter: The image I_1 acts as a virtual object for the *tilted* mirror. The object distance (along the principal axis of the mirror) is $u = 25\cos 30^\circ = 25\sqrt{3}/2$ and height of the object is $h = 25\sin 30^\circ = 25/2$. The mirror formula gives the image distance $v = -50\sqrt{3}/(4-\sqrt{3})$. The magnification by the mirror is $m = -v/u = 4/(4-\sqrt{3})$. Thus, the image height is $\mathrm{VI}_2' = mh = 50/(4-\sqrt{3})$. In right angled triangle PVI_2', angle $\theta = 30^\circ$ and distance $\mathrm{PI}_2' = 100/(4-\sqrt{3})$ cm.

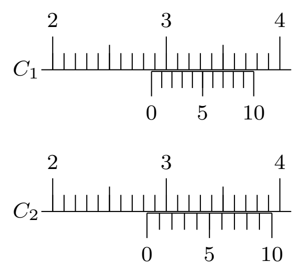


The x and y coordinates of the image I_2' are given by

$$x = 50 - \frac{100}{4-\sqrt{3}}\cos 60^\circ = 50\left(1 - \frac{1}{4-\sqrt{3}}\right) = 28 \text{ cm},$$

$$y = \frac{100}{4-\sqrt{3}}\sin 60^\circ = \frac{50\sqrt{3}}{4-\sqrt{3}} = 38 \text{ cm}.$$

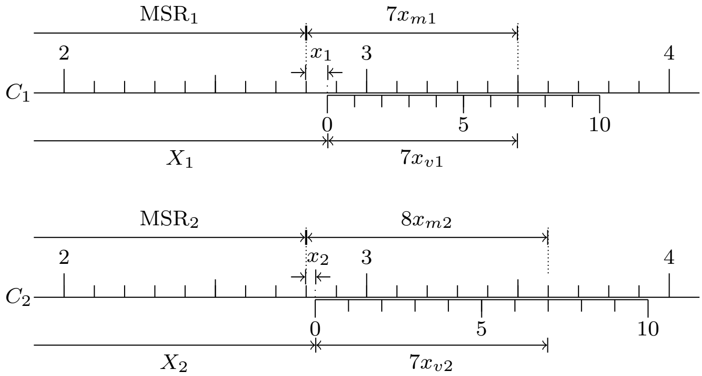
Question 6. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2, respectively are



(2016)

(A) 2.85 and 2.82 (B) 2.87 and 2.83
(C) 2.87 and 2.86 (D) 2.87 and 2.87

Solution. In both calipers C_1 and C_2, 1 cm is divided into 10 equal divisions on the main scale. Thus, 1 division on the main scale is equal to $x_{m1} = x_{m2} = 1\text{ cm}/10 = 0.1$ cm. In calipers C_1, 10 equal divisions on the Vernier scale are equal to 9 main scale divisions. Thus, 1 division on the Vernier scale of C_1 is equal to $x_{v1} = 9x_{m1}/10 = 0.09$ cm. In calipers C_2, 10 equal divisions on the Vernier scale are equal to 11 main scale divisions. Thus, 1 division on the Vernier scale of C_2 is equal to $x_{v2} = 11x_{m2}/10 = 0.11$ cm.



Let main scale reading be MSR and v^{th} division of the Vernier scale coincides with m^{th} division of the main scale (m is counted beyond MSR). The value measured by this calipers is

$$X = \text{MSR} + x = \text{MSR} + mx_m - vx_v. \qquad (1)$$

In calipers C_1, $\text{MSR}_1 = 2.8$ cm, $m_1 = 7$ and $v_1 = 7$ and in calipers C_2, $\text{MSR}_2 = 2.8$ cm, $m_2 = 8$ and $v_2 = 7$. Substitute these values in equation (1) to get

$$X_1 = \text{MSR}_1 + m_1x_{m1} - v_1x_{v1} = 2.8 + 7(0.1) - 7(0.09) = 2.87 \text{ cm}.$$
$$X_2 = \text{MSR}_2 + m_2x_{m2} - v_2x_{v2} = 2.8 + 8(0.1) - 7(0.11) = 2.83 \text{ cm}.$$

The Vernier calipers are generally of type C_1 having $m = v$ and least count $\text{LC} = x_m - x_v$. For these calipers, equation (1) gives $X = \text{MSR} + v \times \text{LC}$.

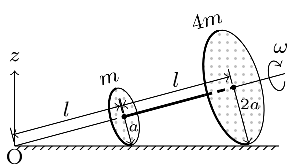
SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:+1	If darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:0	If none of the bubbles is darkened.
Negative Marks	:-2	In all other cases.

- For example, if (a), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in −2 marks, as a wrong option is also darkened.

Question 7. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true? (2016)



(A) The center of mass of the assembly rotates about the z-axis with an angular speed $\omega/5$.

(B) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$.

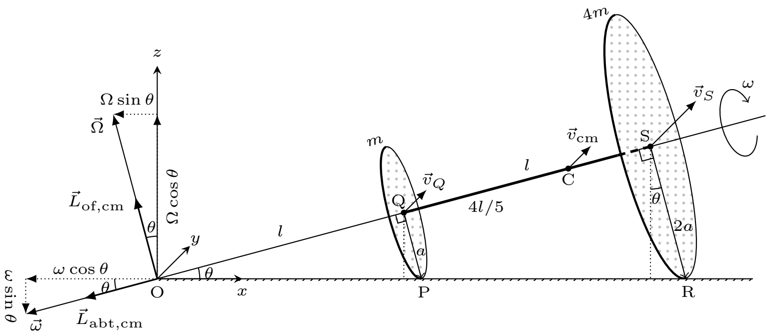
(C) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$.

(D) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$.

Solution. Let x, y, z be the axes as shown in the figure. Without any loss of generality, we can analyse the problem, when points on the disc which are in contact with the ground, lie on the x-axis. As discs are rolling without slipping, the points on the discs which are in contact with the ground, are at rest i.e., $\vec{v}_P = \vec{0}$ and $\vec{v}_R = \vec{0}$. The angular velocity of the discs about an axis joining their centres Q and S is $\vec{\omega}$. The velocities of the points Q and

S are

$$\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{\text{PQ}}$$
$$= (-\omega \cos\theta\, \hat{\imath} - \omega \sin\theta\, \hat{k}) \times (-a \sin\theta\, \hat{\imath} + a \cos\theta\, \hat{k}) = \omega a\, \hat{\jmath}, \quad (1)$$
$$\vec{v}_S = \vec{v}_R + \vec{\omega} \times \vec{\text{RS}}$$
$$= (-\omega \cos\theta\, \hat{\imath} - \omega \sin\theta\, \hat{k}) \times (-2a \sin\theta\, \hat{\imath} + 2a \cos\theta\, \hat{k}) = 2\omega a\, \hat{\jmath}. \quad (2)$$



The points Q and S are located on a rigid rod connecting two discs. The discs rotate (without slipping) with an angular velocity $\vec{\omega}$ about the axis QS. The axis QS itself rotates with an angular velocity $\vec{\Omega}$. Note that the direction of $\vec{\Omega}$ is perpendicular to axis QS. If it is not so then $\vec{\Omega}$ will have a non-zero component along QS. This component will make the discs to rotate faster or slower, which will violate the no-slip condition we have used in equations (1) and (2). Mathematically, $\vec{\Omega} \cdot \vec{\text{QS}} = 0$ i.e., $(\Omega_x \hat{\imath} + \Omega_y \hat{\jmath} + \Omega_z \hat{k}) \cdot (l \cos\theta \hat{\imath} + l \sin\theta \hat{k}) = 0$ which gives

$$\Omega_x l \cos\theta + \Omega_z l \sin\theta = 0. \quad (3)$$

The velocities of Q and S are related by $\vec{v}_S = \vec{v}_Q + \vec{\Omega} \times \vec{\text{QS}}$ which gives $\Omega_y = 0$ and

$$-\Omega_x l \sin\theta + \Omega_z l \cos\theta = \omega a. \quad (4)$$

Solve equations (3) and (4) to get

$$\Omega_x = -(\omega a/l) \sin\theta = -\omega/(5\sqrt{24}), \quad (5)$$
$$\Omega_z = (\omega a/l) \cos\theta = \omega/5, \quad (6)$$

where we have used $l = \sqrt{24}a$, $\cos\theta = \sqrt{24}/5$ and $\sin\theta = 1/5$. Thus, the angular velocity $\vec{\Omega} = (\omega/\sqrt{24})(-\sin\theta \hat{\imath} + \cos\theta \hat{k})$.

The distance of the centre of mass of the system (C) from the origin O and its position vector are given by

$$l_{\text{cm}} = \frac{ml + 4m(2l)}{m + 4m} = \frac{9}{5}l, \qquad \vec{r}_{\text{cm}} = \frac{9}{5}l(\cos\theta \hat{\imath} + \sin\theta \hat{k}) \quad (7)$$

The velocity and linear momentum of the centre of mass C are

$$\vec{v}_{\rm cm} = \frac{m\vec{v}_Q + 4m\vec{v}_S}{m+4m} = \frac{9}{5}\omega a\,\hat{\jmath}, \quad \vec{p}_{\rm cm} = (m+4m)\vec{v}_{\rm cm} = 9m\omega a\,\hat{\jmath}. \tag{8}$$

The angular momentum of the centre of mass about the origin is given by

$$\vec{L}_{\rm of\ cm} = \vec{r}_{\rm cm} \times \vec{p}_{\rm cm} = \frac{81\sqrt{24}}{5} m\omega a^2(-\sin\theta\hat{\imath} + \cos\theta\hat{k}). \tag{9}$$

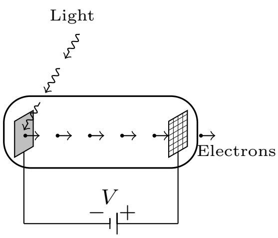
The angular momentum of the system about the centre of mass (it is the angular momentum of the system due to rotation of the discs about the axis QS) is given by

$$\begin{aligned}\vec{L}_{\rm about\ cm} &= I_{\rm cm}\vec{\omega} = \left(\frac{1}{2}ma^2 + \frac{1}{2}(4m)(2a)^2\right)\vec{\omega} \\ &= \frac{17}{2}m\omega a^2(-\cos\theta\hat{\imath} - \sin\theta\hat{k}).\end{aligned} \tag{10}$$

Total angular momentum of the system is given by

$$\vec{L} = \vec{L}_{\rm of\ cm} + \vec{L}_{\rm about\ cm} = \left(-\frac{247\sqrt{6}}{25}\hat{\imath} + \frac{3803}{50}\hat{k}\right) m\omega a^2. \tag{11}$$

Question 8. Light of wavelength $\lambda_{\rm ph}$ falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electron passing through the anode is λ_e, which of the following statement(s) is(are) true?



(2016)

(A) λ_e decreases with increase in ϕ and $\lambda_{\rm ph}$.
(B) λ_e is approximately halved, if d is doubled.
(C) For large potential difference ($V \gg \phi/e$), λ_e is approximately halved if V is made four times.
(D) λ_e increases at the same rate as $\lambda_{\rm ph}$ for $\lambda_{\rm ph} < hc/\phi$.

Solution. In photo-electric effect, the maximum kinetic energy of the photo-electron ejected at the cathode, is given by

$$K_{\rm max,c} = hc/\lambda_{\rm ph} - \phi, \tag{1}$$

where $\lambda_{\rm ph}$ is the wavelength of the ejected photo-electron and ϕ is the work function of the material. The ejected photo-electrons are accelerated from the cathode to the anode by a potential V. Thus, the maximum kinetic energy of the photo-electron at the anode is

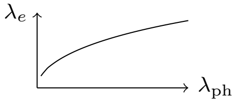
$$K_{\text{max,a}} = K_{\text{max,c}} + eV = hc/\lambda_{\text{ph}} - \phi + eV. \tag{2}$$

The linear momentum of the electron of mass m and kinetic energy K is given by $p = \sqrt{2mK}$. Thus, the minimum de-Broglie wavelength of the electron at the anode is

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mK_{\text{max,a}}}} = \frac{h}{\sqrt{2m(hc/\lambda_{\text{ph}} - \phi + eV)}}. \tag{3}$$

The wavelength λ_e increases with increase in ϕ and λ_{ph} as denominator of equation (3) decreases with increase in ϕ and λ_{ph}. Also, λ_e is independent of d. The order of magnitudes of hc/λ_{ph} and ϕ is almost same. Thus, if $V \gg \phi/e$ then equation (3) reduces to $\lambda_e \approx h/\sqrt{2meV}$. Hence, λ_e is approximately halved if V is made four times.

The equation (3) is not linear in λ_{ph} and λ_e. Thus, the rates of increase of λ_e and λ_{ph} will be different. This can be verified by differentiating equation (3). Show that the slope $\mathrm{d}\lambda_e/\mathrm{d}\lambda_{\text{ph}}$ is different at different values of λ_{ph}.



Question 9. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true? (2016)

(A) The error in the measurement of r is 10%.

(B) The error in the measurement of T is 3.57%.

(C) The error in the measurement of T is 2%.

(D) The error in the determined value of g is 11%.

Solution. The relative percentage error in the measurement of r is given by

$$e_r = \frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%.$$

The mean value of time period is given by

$$\bar{T} = \frac{\sum_1^5 T_i}{5} = \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5} = 0.556 \text{ s} \approx 0.56 \text{ s},$$

where we have truncated at 2 significant figures. The absolute error and the relative percentage error in the measurement of T are given by

$$\Delta T = \frac{\sum_1^5 |T_i - \bar{T}|}{5} = \frac{0.04 + 0 + 0.01 + 0.02 + 0.03}{5} = 0.02.$$

$$e_T = \frac{\Delta T}{\bar{T}} \times 100 = \frac{0.02}{0.56} \times 100 = 3.57\%.$$

Given equation for time period can be written as $g = \frac{28\pi^2}{5}\frac{R-r}{T^2}$. Differentiate it to get the relative error in g as

$$\frac{\Delta g}{g} = \frac{\Delta(R-r)}{R-r} + 2\frac{\Delta T}{T} = \frac{\Delta R + \Delta r}{R-r} + 2\frac{\Delta T}{T}$$

$$= \frac{1+1}{60-10} + 2\frac{0.02}{0.56} = 0.11 = 11\%.$$

It is interesting to note that the time period $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$ is same as the time period of a small spherical ball of mass m and radius r when it rolls without slipping on a rough concave surface of large radius R. The readers are encouraged to derive it.

Question 10. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_c < R/2$, which of the following statement(s) about any one of the galvanometers is(are) true? (2016)

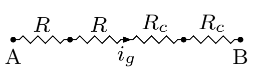
(A) The maximum voltage range is obtained when all the components are connected in series.

(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer.

(C) The maximum current range is obtained when all the components are connected in parallel.

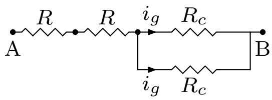
(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

Solution. Let i_g be the current that gives full deflection of the galvanometer. When all components are connected in series (see figure), effective resistance of the circuit is $R_e = 2R + 2R_c$ and maximum current allowed in the circuit is i_g. Thus, the voltage between A and B is



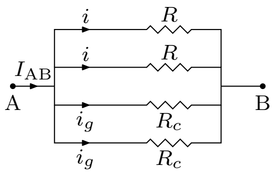
$$V_{\text{AB}} = i_g R_e = 2i_g(R + R_c).$$

Consider the case when two resistors and one galvanometer are connected in series and the second galvanometer is connected in parallel (see figure). The maximum current through each galvanometer is i_g and the maximum current through the resistors is $2i_g$. Apply Kirchhoff's law to get the voltage between A and B as



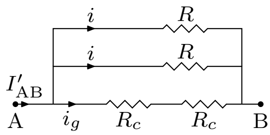
$$\begin{aligned} V'_{\mathrm{AB}} &= 2i_gR + 2i_gR + i_gR_c = 2i_g(R+R_c) + 2i_g(R - R_c/2) \\ &= V_{\mathrm{AB}} + 2i_g(R - R_c/2) > V_{\mathrm{AB}} \qquad (\because R_c < R/2). \end{aligned}$$

Consider the case when all four components are connected in parallel (see figure). Let i and i_g be the currents through the resistors and the galvanometers. By Kirchhoff's law, $iR = i_gR_c$, which gives $i = i_gR_c/R$. The current between A and B is



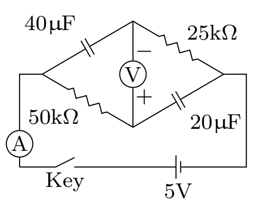
$$I_{\mathrm{AB}} = 2i + 2i_g = 2i_g(1 + R_c/R).$$

Consider the case when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors (see figure). By Kirchhoff's law, $iR = i_g(2R_c)$, which gives $i = 2i_gR_c/R$. The current between A and B is



$$\begin{aligned} I'_{\mathrm{AB}} &= i_g + 2i = i_g + 2i_g(2R_c/R) = 2i_g(1 + R_c/R) - (1 - 2R_c/R)i_g \\ &= I_{\mathrm{AB}} - (1 - 2R_c/R)i_g < I_{\mathrm{AB}} \qquad (\because R_c < R/2). \end{aligned}$$

Question 11. In the circuit shown in the figure, the key is pressed at time $t = 0$. Which of the following statement(s) is(are) true? (2016)



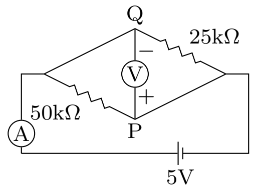
(A) The voltmeter displays −5 V as soon as the key is pressed, and displays +5 V after a long time.

(B) The voltmeter will display 0 V at time $t = \ln 2$ seconds.

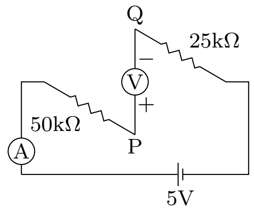
(C) The current in the ammeter becomes $1/e$ of the initial value after 1 second.

(D) The current in the ammeter becomes zero after a long time.

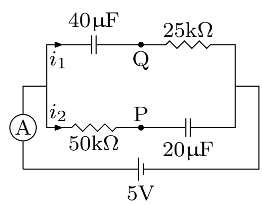
Solution. The impedance (effective resistance) of the capacitors is zero immediately after the key is pressed ($t \to 0^+$). Thus, capacitors will act as short circuit elements (see figure). Thus, potential at the node Q is $V_Q = 5$ V and potential at the node P is $V_P = 0$ V. The voltmeter will display $V = V_P - V_Q = 0 - 5 = -5$ V.



The impedance of the capacitors is infinite in steady state ($t \to \infty$) and they act as open circuit elements (see figure). We assume voltmeter to be ideal with infinite resistance. Thus, no current flows through the circuit and hence ammeter will read zero. Also, potential at the node Q is $V_Q = 0$ V and potential at the node P is $V_P = 5$ V. The voltmeter will display $V = V_P - V_Q = 5 - 0 = 5$ V.



The capacitor $C_1 = 40$ µF is connected across a $V = 5$ V battery by a series resistance $R_1 = 25$ kΩ. Similarly, the capacitor $C_2 = 20$ µF is connected across the same battery by a series resistance $R_2 = 50$ kΩ. We can remove voltmeter from the circuit for current calculation as it has infinite resistance. This circuit is used to charge both the capacitors. The currents through the capacitors at time t (charging of the capacitor) are given by



$$i_1 = (V/R_1)e^{-\frac{t}{R_1C_1}} = 200e^{-t} \text{ µA}, \tag{1}$$
$$i_2 = (V/R_2)e^{-\frac{t}{R_2C_2}} = 125e^{-t} \text{ µA}. \tag{2}$$

From equations (1) and (2), the current through the ammeter at time $t = 0$ s is $i_0 = i_1 + i_2 = 325$ µA. The current through the ammeter at time $t = 1$ s is $i = i_1 + i_2 = 200e^{-1} + 125e^{-1} = 325/e = i_0/e$.

The potentials at points P and Q at a time t are given by

$$V_P = V - i_2R_2 = 5(1 - e^{-t}), \tag{3}$$
$$V_Q = 0 + i_2R_1 = 5e^{-t}. \tag{4}$$

From equations (3) and (4), the voltmeter at time $t = \ln 2$ will display $V = V_p - V_Q = 5 - 10e^{-\ln 2} = 0$ V.

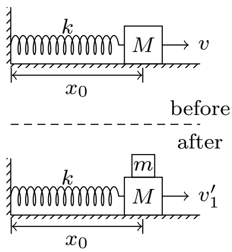
Question 12. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (1) when the block is at x_0; and (2) when the block is at $x = x_0 + A$. In both the cases, a particle with mass $m\,(< M)$ is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? (2016)

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.
(B) The final time period of oscillation in both the cases is same.
(C) The total energy decreases in both the cases.
(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

Solution. The angular frequency of the spring mass system depends on the spring constant and the attached mass. Initial angular frequency of the system (ω), final angular frequency of the system in case 1 (ω_1'), and final angular frequency of the system in case 2 (ω_2') are given by

$$\omega = \sqrt{k/M}, \qquad \omega_1' = \sqrt{k/(M+m)}, \qquad \omega_2' = \sqrt{k/(M+m)}. \qquad (1)$$

The final time period of oscillation ($T = 2\pi/\omega$) in both the cases is same because $\omega_1' = \omega_2'$. For SHM in horizontal plane, mean position x_0 is equal to the natural length of the spring. Thus, the mean position x_0 remains same in both the cases.



Let the initial amplitude of SHM be A, final amplitude in case 1 be A_1', and final amplitude in case 2 be A_2'. In case 1, the speed of the mass M at the mean position just before mass m is softly placed over it is $v = \omega A$. Let the speed just after mass m is placed over it be v_1'. The linear momentum of the system along the direction of motion is conserved because there is no external force on it in the direction of motion. Apply conservation of linear momentum, $Mv = (M+m)v_1'$, to get

$$v_1' = Mv/(M+m) = M\omega A/(M+m). \qquad (2)$$

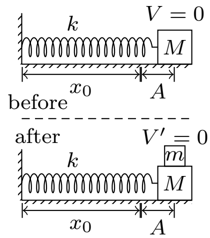
From equation (2), the instantaneous speed at the mean position is decreased i.e., $v_1' < v$. The speed at the mean position v_1' is related to the amplitude A_1' by $v_1' = A_1'\omega_1'$. Substitute for v_1' and ω_1' to get

$$A_1' = \frac{v_1'}{\omega_1'} = \frac{MA}{M+m}\frac{\sqrt{k/M}}{\sqrt{k/(M+m)}} = A\sqrt{\frac{M}{M+m}}. \qquad (3)$$

Let T be the initial total energy, T_1' be the final total energy in case 1, and T_2' be the final total energy in case 2. The final energy in case 1 is given by

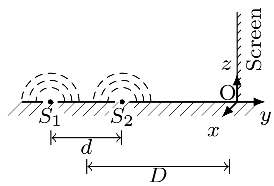
$$T_1' = \frac{1}{2}k{A_1'}^2 = \frac{1}{2}kA^2\frac{M}{M+m} = T\frac{M}{M+m}. \quad \text{(using equation (3))}.$$

In case 2, the mass m is placed over the block when it is at extreme position $x = x_0 + A$. Thus, velocity of the block just before m is placed over it is zero. The conservation of linear momentum gives that the velocity of the combined masses just after m is placed over the block is also zero. Hence the position $x = x_0 + A$ remains as the extreme position for SHM of combined masses. Thus, the amplitude of the SHM remains unchanged in case 2 i.e., $A_2' = A$. The total energy of the system also remains unchanged in case 2 i.e., $T_2' = \frac{1}{2}k{A_2'}^2 = \frac{1}{2}kA^2 = T$. The instantaneous speed at the mean position in case 2 is given by



$$v_2' = \omega_2' A_2' = \sqrt{\frac{k}{M+m}}A = \sqrt{\frac{M}{M+m}}\omega A = \frac{M}{M+m}v.$$

Question 13. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources is $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is(are) true of the intensity pattern on the screen? (2016)



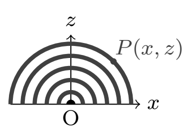
(A) Straight bright and dark bands parallel to the x-axis.
(B) The region very close to the point O will be dark.
(C) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction.
(D) Semi circular bright and dark bands centered at point O.

Solution. The waves from the sources S_1 and S_2 interfere at the origin O. The path difference between these waves is

$$\Delta = \mathrm{S_1O} - \mathrm{S_2O} = d = 0.6003\ \text{mm} = 2001(\lambda/2),$$

where $\lambda = 600$ nm. The path difference at O is odd integral multiple of $\lambda/2$. Thus there will be dark fringe at O.

It can be easily seen that the path difference is same for all points on the screen which lie on a circle centered at O. It can be proved by taking a point P (x, z) on the screen. The path difference at P is given by



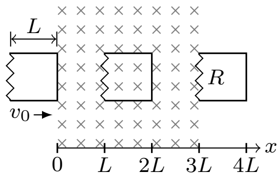
$$\Delta = \mathrm{S_1P} - \mathrm{S_2P} = \sqrt{x^2 + z^2 + (D + \tfrac{d}{2})^2} - \sqrt{x^2 + z^2 + (D - \tfrac{d}{2})^2}. \quad (1)$$

Simplify equation (1) to get (you need to square it twice)

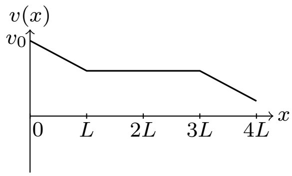
$$x^2 + z^2 = \frac{D^2 d^2}{\Delta^2} + \frac{\Delta^2}{4} - D^2 - \frac{d^2}{4} = r^2, \tag{2}$$

which is an equation of a circle with centre O and radius r. Note that Δ is constant at a fringe. Since the screen covers upper half of the x-z plane, the student will see semi-circular bright and dark bands centred at point O.

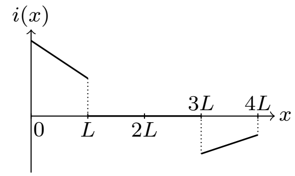
Question 14. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $i(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively as a function of x. Counter-clockwise current is taken as positive. Which of the following schematic plot(s) is(are) correct? [Ignore gravity.] (2016)



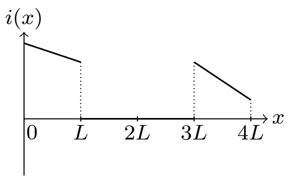
(A)



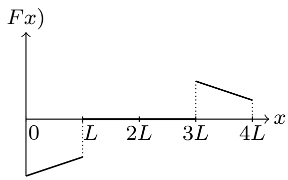
(B)



(C)



(D)



Solution. Let the right edge of the loop be at $x\,(< L)$ i.e., the right edge of the loop is inside the magnetic field B_0 and the left edge is outside B_0. Let the velocity of the loop at this instant be v. By Faraday's law, the induced *emf* in the loop is $e = B_0 L v$ and induced current in the loop is

$$i = e/R = B_0 L v / R. \tag{1}$$

By Lenz's law, the induced current should oppose the increase in magnetic flux through the loop. Thus, the magnetic field due to the induced current should be coming out of the paper and hence current in the loop should be counterclockwise (see figure). The magnetic force acting on the right edge is $\vec{F} = i\vec{L} \times \vec{B}_0 = -iLB_0\,\hat{\imath}$ (towards left). Let m be the mass of the loop. Apply Newton's second law to get

$$F = m\mathrm{d}v/\mathrm{d}t = mv\mathrm{d}v/\mathrm{d}x = -iLB_0 = -(B_0^2 L^2/R)v, \tag{2}$$

where we have used equation (1) to replace i. Integrate equation (2) from $x = 0$ to $x\,(< L)$ to get

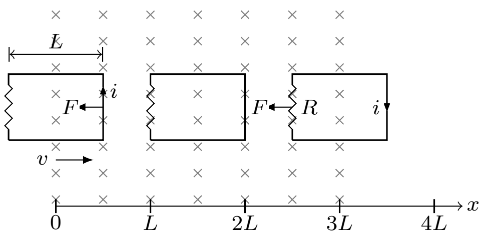
$$v(x) = v_0 - \frac{B_0^2 L^2}{mR} x. \tag{3}$$

Substitute $v(x)$ from equation (3) in equations (1) and (2) to get

$$i(x) = \frac{B_0 L}{R}\left(v_0 - \frac{B_0^2 L^2}{mR} x\right), \tag{4}$$

$$F(x) = -\frac{B_0^2 L^2}{R}\left(v_0 - \frac{B_0^2 L^2}{mR} x\right). \tag{5}$$

Thus, the velocity $v(x)$ and the current $i(x)$ decrease linearly with x but the force $F(x)$ increases linearly with x.



Consider the case when loop is completely inside the magnetic filed i.e., $L < x < 3L$. The magnetic flux through the loop is constant. Thus, the induced *emf* $e = 0$, induced current $i = 0$, and magnetic force $F = 0$ (as $i = 0$). The velocity of the loop remains constant (as $F = 0$) at its value at $x = L$ i.e., $v = v_0 - B_0^2 L^3/(mR)$.

Now, consider the case when the right edge of the loop is at $x\,(> 3L)$. Let the loop velocity at this instant be v. The induced *emf* in the loop is $e = B_0 L v$ and induced current is $i = B_0 L v/R$. By Lenz's law, the induced current should oppose the decrease in magnetic flux. Thus, the magnetic field due to the induced current should be into the paper and hence induced current is clockwise. The magnetic force acting on the left edge is $\vec{F} = i\vec{L} \times \vec{B}_0 = -(B_0^2 L^2/R) v\,\hat{\imath}$. Apply Newton's second law and integrate from $x = 3L$ to $x\,(< 4L)$ to get

$$v(x) = v_0 - \frac{B_0^2 L^3}{mR} - \frac{B_0^2 L^2}{mR}(x - 3L). \tag{6}$$

Substitute $v(x)$ from equation (6) into the expressions for the induced current and the force to get

$$i(x) = \frac{B_0 L}{R}\left(v_0 - \frac{B_0^2 L^3}{mR} - \frac{B_0^2 L^2}{mR}(x - 3L)\right), \tag{7}$$

$$F(x) = -\frac{B_0^2 L^2}{R}\left(v_0 - \frac{B_0^2 L^3}{mR} - \frac{B_0^2 L^2}{mR}(x - 3L)\right). \tag{8}$$

Thus, the velocity $v(x)$ and the current $i(x)$ decrease linearly with x but the force $F(x)$ increases linearly with x. The readers are encouraged to plot $v(x)$, $i(x)$, and $F(x)$. What is the minimum value of v_0 for the loop to come out of the magnetic field?

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks :+3 If only the bubble corresponding to all the correct option is darkened.

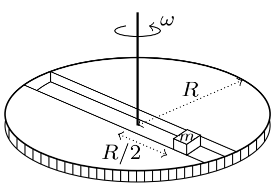
  Zero Marks:0 In all other cases.

Paragraph for Question 15-16

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{\text{rot}}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{\text{in}}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(\vec{v}_{\text{rot}} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

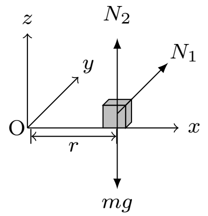
where $\vec{v}_{\text{rot}}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.



Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ($\vec{\omega} = \omega\hat{k}$). A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\,\hat{\imath}$ at $t = 0$ and is constrained to move only along the slot. (2016)

Question 15. The distance r of the block at time t is
(A) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$ (B) $\frac{R}{2}\cos\omega t$ (C) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (D) $\frac{R}{2}\cos 2\omega t$

Solution. We will solve this problem in a rotating frame (x, y, z) attached to the disc. Let the block is at a distance r from the centre of the disc. The real forces $(\vec{F}_{\text{in}})$ acting on the block are its weight mg, normal reaction N_2 from the bottom surface of the groove, and normal reaction N_1 from the side surface of the groove (see figure). Since the groove is smooth, there is no frictional force along the groove. Thus, the total force on the block is



$$\vec{F}_{\text{in}} = N_1\,\hat{\jmath} + (N_2 - mg)\,\hat{k}. \tag{1}$$

The force in the rotating frame is given by

$$\vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(\vec{v}_{\text{rot}} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}. \tag{2}$$

In the rotating frame (x, y, z), position vector of the block is $\vec{r} = r\,\hat{\imath}$, velocity of the block is $\vec{v}_{\text{rot}} = (\mathrm{d}r/\mathrm{d}t)\,\hat{\imath}$ (because the block is constrained to move along the groove), acceleration of the block is $\vec{a}_{\text{rot}} = \mathrm{d}^2r/\mathrm{d}t^2\,\hat{\imath}$, (because the block is constrained to move along the groove), and angular velocity of the frame (disc) is $\vec{\omega} = \omega\,\hat{k}$. Newton's second law in rotating frame is written as $\vec{F}_{\text{rot}} = m\vec{a}_{\text{rot}}$. Substitute these parameters in equation (2) to get

$$\begin{aligned} m\frac{\mathrm{d}^2r}{\mathrm{d}t^2}\,\hat{\imath} &= (N_1\,\hat{\jmath} + (N_2 - mg)\,\hat{k}) + 2m(\mathrm{d}r/\mathrm{d}t\,\hat{\imath} \times \omega\,\hat{k}) \\ &\quad + m(\omega\,\hat{k} \times r\,\hat{\imath}) \times \omega\,\hat{k} \\ &= m\omega^2 r\,\hat{\imath} + (N_1 - 2m\omega\,\mathrm{d}r/\mathrm{d}t)\,\hat{\jmath} + (N_2 - mg)\,\hat{k}. \end{aligned} \tag{3}$$

Compare x, y and z components on two sides of equation (3) to get

$$\mathrm{d}^2r/\mathrm{d}t^2 = \omega^2 r, \tag{4}$$

$$N_1 = 2m\omega\,\mathrm{d}r/\mathrm{d}t, \tag{5}$$

$$N_2 = mg. \tag{6}$$

The equation (4) is a linear differential equation of second order. In higher classes, you will learn various techniques to solve this type of equations. It is easy to show (by substitution) that $r(t) = \frac{R}{4}(e^{\omega t} + e^{-\omega t})$ is the only solution from the given options that satisfies equation (4). Also, it satisfies the initial condition i.e., $r = R/2$ at $t = 0$.

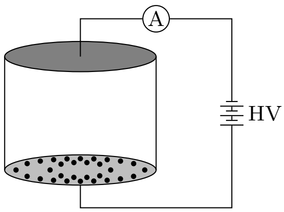
Question 16. The net reaction of the disc on the block is
(A) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\,\hat{\jmath} + mg\,\hat{k}$
(B) $\frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\,\hat{\jmath} + mg\,\hat{k}$
(C) $-m\omega^2 R\cos\omega t\,\hat{\jmath} - mg\,\hat{k}$
(D) $m\omega^2 R\sin\omega t\,\hat{\jmath} - mg\,\hat{k}$

Solution. The net reaction on the block is

$$\begin{aligned}\vec{N} &= N_1\,\hat{\jmath} + N_2\hat{k} = -2m\omega\,\mathrm{d}r/\mathrm{d}t\,\hat{\jmath} + mg\,\hat{k} \\ &= 2m\omega\,(R\omega/4)(e^{\omega t} - e^{-\omega t})\,\hat{\jmath} + mg\,\hat{k} \\ &= \tfrac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\,\hat{\jmath} + mg\,\hat{k},\end{aligned}$$

where we have used expressions of N_1, N_2, and $r(t)$ from the previous question.

Paragraph for Question 17-18



Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r \ll h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. [Ignore gravity.] (2016)

Question 17. Which one of the following statements is correct?

(A) The balls will stick to the top plate and remain there.

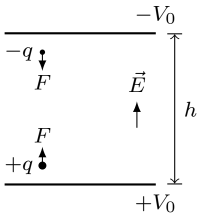
(B) The balls will bounce back to the bottom plate carrying the same charge they went up with.

(C) The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

(D) The balls will execute simple harmonic motion between the two plates.

Solution. The distance between the two plates is h. The potential of the bottom plate is V_0 and that of the top plates is $-V_0$. The electric field between the plates is $E = 2V_0/h$ (directed upwards). The radius of each ball is r ($\ll h$). Let m be the mass and C be the capacitance of each ball.

When the ball touches the bottom plate, it gets a positive charge $q = CV_0$ (we assume that the charge transfer is instantaneous). This positively charged ball experiences an upward force, $F = qE = 2CV_0^2/h$, which accelerates the ball upwards. Since the force is constant, the ball cannot do SHM (for SHM, the force should be proportional to the displacement and directed towards the centre).



When the ball hits the top plate, it transfers the positive charge to the plate and gets negative charge $q = -CV_0$. This negatively charged ball again experience a force $F = qE$ (downward) and starts accelerating downwards. Thus, the ball keeps moving between the bottom and the top plates carrying a charge $+q$ upwards and $-q$ downwards.

Question 18. The average current in the steady state registered by the ammeter in the circuit will be

(A) zero
(B) proportional to the potential V_0
(C) proportional to $V_0^{1/2}$
(D) proportional to V_0^2

Solution. The average current when a charge q moves from the bottom plate to the top plate in time T is $i = q/T$. The ball moves with a uniform acceleration $a = F/m = 2CV_0^2/(mh)$. The distance travelled in time T is

$$h = \frac{1}{2}aT^2 = \frac{1}{2}\,\frac{2CV_0^2}{mh}\,T^2, \qquad \text{i.e.,} \qquad T = \frac{h}{V_0}\sqrt{\frac{m}{C}}.$$

The average current carried by each ball is

$$i = \frac{q}{T} = \frac{C}{h}\sqrt{\frac{C}{m}}V_0^2.$$