IIT JEE (Advanced) 2016 Paper 1: PHYSICS

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:0	If none of the bubbles is darkened.
Negative Marks	:-1	In all other cases.

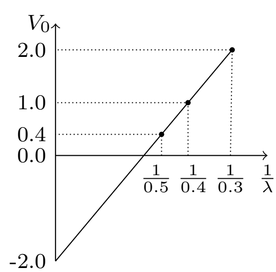
Question 1. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m/s and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J-s) found from such an experiment is (2016)
(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Solution. In photoelectric effect, the stopping potential (V_0) is related to the wavelength of incident radiation by

$$eV_0 = hc/\lambda - \phi, \qquad (1)$$



where ϕ is the work function of the metal. From equation (1), $1/\lambda$ *versus* V_0 graph is a straight line with a slope hc/e and intercept $-\phi/e$ on V_0 axis. Substitute the values of V_0 and λ in equation (1) to get three equations in two unknowns, hc/e and ϕ/e. Solve any two of the three equations to get $h = 6.4 \times 10^{-34}$ J-s and $\phi = 2.0$ eV. The readers are encouraged to find h and ϕ if second reading is erroneously measured as (0.4 μm, 1.1 V) instead of (0.4 μm, 1.0 V).

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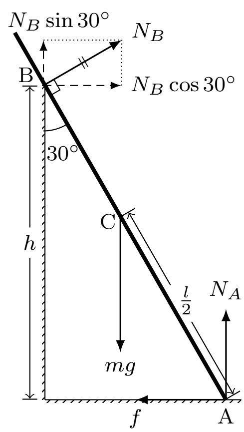
Question 2. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h\,(< l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are $[\,g = 10\ \mathrm{m/s^2}\,]$ (2016)

(A) $\frac{h}{l} = \frac{\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N (B) $\frac{h}{l} = \frac{3}{16}$, $f = \frac{16\sqrt{3}}{3}$ N
(C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}$, $f = \frac{8\sqrt{3}}{3}$ N (D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

Solution. The forces acting on the stick are its weight mg at the centre of mass C, normal reaction N_A at the contact point A due to the floor, frictional force f at the point A due to the rough floor, and the normal reaction N_B at the contact point B due to the wall. Since stick is uniform, its centre of mass C lies at the middle point i.e., at a distance $l/2$ from the end A. It is given that the direction of N_B is perpendicular to the stick and $N_B = N_A$. Resolve N_B in the horizontal and the vertical directions (see figure). Since the stick is in equilibrium, the net forces on the stick in the horizontal and the vertical directions are zero i.e.,



$$N_A + N_B \sin 30° - mg = 0, \quad (1)$$
$$N_B \cos 30° - f = 0. \quad (2)$$

Also, in equilibrium, net torque on the stick about any point should be zero. The net torque on the stick about the point A is

$$mg(l/2)\cos 60° - (N_B \cos 30°)h - (N_B \sin 30°)h \tan 30° = 0. \quad (3)$$

Substitute $N_B = N_A$ and solve equations (1)–(3) to get

$$\frac{h}{l} = \frac{3\sqrt{3}}{16}, \quad \text{and} \quad f = \frac{16\sqrt{3}}{3}\ \text{N}.$$

The readers are encouraged to solve this problem by taking the torque about the point B or C.

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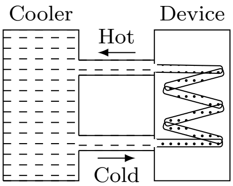
Question 3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30 °C and the entire stored 120 litres of water is initially cooled to 10 °C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is [Specific heat of water is 4.2 kJ kg^{-1}K^{-1} and the density of water is 1000 kg/m^3.] (2016)



(A) 1600 (B) 2067 (C) 2533 (D) 3933

Solution. Let $P_d = 3$ kW be the power generated by the device. Let the water temperature increases from T to $T + \Delta T$ in a time interval Δt. In this time interval, the energy generated by the device is $P_d \Delta t$, the energy pumped out by the cooler is $P\Delta t$ and the energy used to increase the temperature of water is $mS\Delta T$. The energy of the system is conserved because it is thermally insulated i.e.,

$$P_d \Delta t = P\Delta t + mS\Delta T, \tag{1}$$

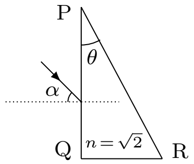
which gives

$$P = P_d - \frac{mS\Delta T}{\Delta t} = P_d - \frac{\rho V S\Delta T}{\Delta t}. \tag{2}$$

The minimum value of P occurs when ΔT attains its maximum allowed value i.e., $\Delta T = 30\ °\text{C} - 10\ °\text{C} = 20\ °\text{C}$. Substitute the values of various parameters in equation (2) to get

$$P = 3 \times 10^3 - \frac{(1000)\,(120 \times 10^{-3})\,(4.2 \times 10^3)\,(20)}{3 \times 3600} = 2067 \text{ W}.$$

Question 4. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is (2016)



(A) 15° (B) 22.5° (C) 30° (D) 45°

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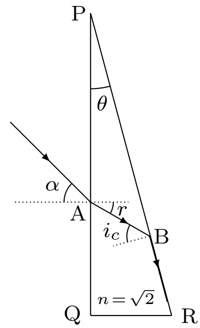
Solution. The ray diagram when a ray strikes the face PR at the critical angle is shown in the figure. This condition occurs when angle of incidence at the face PQ is minimum i.e., $\alpha = 45°$. Apply Snell's law, $\sin\alpha/\sin r = n$, at interface PQ to get the angle of refraction,



$$r = \sin^{-1}\left(\tfrac{\sin\alpha}{n}\right) = \sin^{-1}\left(\tfrac{\sin 45°}{\sqrt{2}}\right) = 30°.$$

The critical angle for total internal reflection at the prism-air interface PR is given by

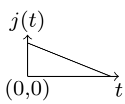
$$i_c = \sin^{-1}\left(\tfrac{1}{n}\right) = \sin^{-1}\left(\tfrac{1}{\sqrt{2}}\right) = 45°.$$

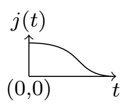
In triangle PAB,

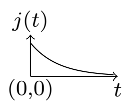
$$\theta + (90° + r) + (90° - i_c) = 180°.$$

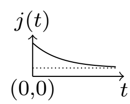
Substitute the values of r and i_c and solve to get $\theta = 15°$.

Question 5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t = 0$, the space inside the cylinder is filled with a material of permittivity ϵ and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of the current density $j(t)$ at any point in the material? (2016)

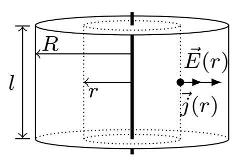
(A) 

(B) 

(C) 

(D) 

Solution. Let the charge per unit length on the axis be $\lambda(t)$ at a time t. The electric field due to this line charge at a radial distance r is given by



$$\vec{E}(r,t) = \frac{\lambda(t)}{2\pi\epsilon r}\hat{r}. \quad (1)$$

The free charges inside the conductor start moving radially due to the presence of electric field. The current density at a point is defined as the current flowing across a unit area placed perpendicular to the direction of current flow. The current density at a point is related to the electric field at that point by Ohm's law i.e.,

$$\vec{j}(r,t) = \sigma\vec{E}(r,t), \quad (2)$$

where σ is the electrical conductivity of the conductor. The current through a cylindrical shell of radius r and length l is given by

$$I = \int_{\text{surface}} \vec{j}(r,t) \cdot \mathrm{d}\vec{A} = j(r,t)(2\pi r l). \tag{3}$$

By conservation of charge, the current I is equal to the rate of decrease of charge q on axial line segment of length l i.e.,

$$I = -\frac{\mathrm{d}q}{\mathrm{d}t} = -l\frac{\mathrm{d}\lambda(t)}{\mathrm{d}t}. \tag{4}$$

From equations (1)–(4)

$$\frac{\mathrm{d}\lambda(t)}{\mathrm{d}t} = -\frac{\sigma\lambda(t)}{\epsilon}. \tag{5}$$

Integrate equation (5) with initial condition $\lambda(t=0) = \lambda_0$ to get

$$\lambda(t) = \lambda_0 \exp e^{-\frac{\sigma t}{\epsilon}}, \tag{6}$$

Substitute $\lambda(t)$ in equation (2) to get

$$\vec{j}(r,t) = \frac{\sigma\lambda_0}{2\pi\epsilon r} e^{-\frac{\sigma t}{\epsilon}}. \tag{7}$$

Note that current density varies as $1/r$ with radial distance r. It decreases exponentially with time and becomes zero as $t \to \infty$. The readers are encouraged to show that equation (2) is equivalent to the popular form of Ohm's law, $V = IR$.

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:+1	If darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.
Zero Marks	:0	If none of the bubbles is darkened.
Negative Marks	:-2	In all other cases.

- For example, if (a), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in −2 marks, as a wrong option is also darkened.

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Question 6. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n \gg 1$. Which of the following statement(s) is(are) true? (2016)

(A) Relative change in the radii of two consecutive orbitals does not depend on Z.
(B) Relative change in the radii of two consecutive orbitals varies as $1/n$.
(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$.
(D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$.

Solution. The radius of n^{th} orbital for a hydrogen-like atom of atomic number Z is given by $r_n = n^2 a_0/Z$, where $a_0 = 0.53$ Å is the Bohr's radius. The relative change in the radii of two consecutive orbitals is

$$\frac{r_{n+1} - r_n}{r_n} = \frac{(n+1)^2 a_0/Z - n^2 a_0/Z}{n^2 a_0/Z} = \frac{2n+1}{n^2} \approx \frac{2}{n} \quad (\because n \gg 1).$$

The energy of the n^{th} orbital is given by $E_n = -13.6\text{Z}^2/\text{n}^2$ eV. The relative change in the energy of two consecutive orbitals is

$$\frac{E_{n+1} - E_n}{E_n} = \frac{-13.6Z^2/(n+1)^2 - (-13.6Z^2/n^2)}{-13.6Z^2/n^2} \approx -\frac{2}{n}. \quad (\because n \gg 1).$$

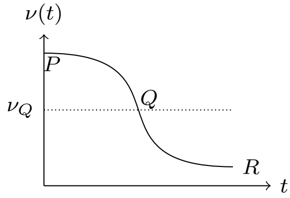
The angular momentum of the n^{th} orbital is given by $L_n = nh/(2\pi)$. The relative change in the angular momentum of two consecutive orbitals is

$$\frac{L_{n+1} - L_n}{L_n} = \frac{(n+1)h/(2\pi) - nh/(2\pi)}{nh/(2\pi)} = \frac{1}{n}.$$

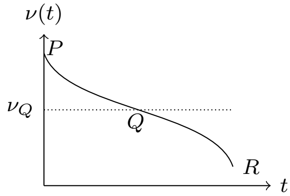
Question 7. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m/s. Which of the following statement(s) is(are) true regarding the sound heard by the person? (2016)

(A) $\nu_P + \nu_R = 2\nu_Q$.
(B) The rate of change in beat frequency is maximum when the car passes through Q.

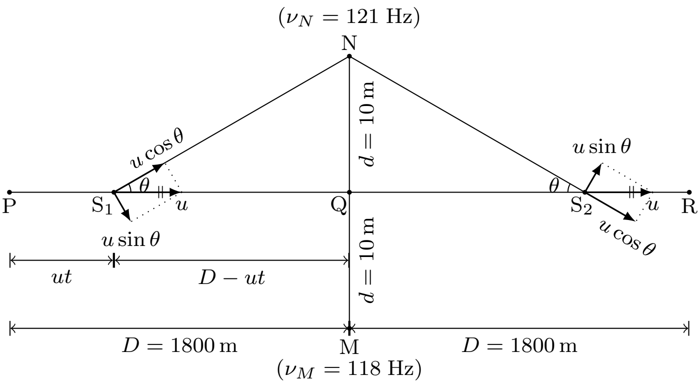
(C) The plot below represents schematically the variation of beat frequency with time.



(D) The plot below represents schematically the variation of beat frequency with time.



Solution. The frequencies of the sources M and N are $\nu_M = 118$ Hz and $\nu_N = 121$ Hz. The distances MQ = QN = $d = 10$ m and PQ = QR = $D = 1800$ m (see figure). The speed of sound in air is $v = 330$ m/s and speed of the car is $u = 60$ km/hr $= 50/3$ m/s. The car will reach Q at time $t_Q = D/u = 108$ s and it will reach R at time $t_R = 2D/u = 216$ s.



Consider the time $t\,(\leq t_Q)$ when car is at S_1 between P and Q. The distance travelled by the car in time t is $\mathrm{PS}_1 = ut$. At this instant, the lines S_1N and S_1M both make angle θ with the velocity vector $\vec{u}$. The component of observer (person sitting in the car) velocity towards the sources N and M is $u_o = u\cos\theta$. The sources N and M are at rest i.e., $u_s = 0$. Apply Doppler's effect equation to get frequencies of the sources N and M heard

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by the observer as

$$\nu'_N = \frac{v+u_o}{v-u_s}\nu_N = \frac{v+u\cos\theta}{v}\nu_N = \left(1+\frac{u}{v}\frac{D-ut}{\sqrt{d^2+(D-ut)^2}}\right)\nu_N,$$
$$\nu'_M = \frac{v+u_o}{v-u_s}\nu_M = \frac{v+u\cos\theta}{v}\nu_M = \left(1+\frac{u}{v}\frac{D-ut}{\sqrt{d^2+(D-ut)^2}}\right)\nu_M.$$

The beat frequency heard by the observer at time $t\,(\leq t_Q)$ is

$$\nu(t) = \nu'_N - \nu'_M = \left(1+\frac{u}{v}\frac{D-ut}{\sqrt{d^2+(D-ut)^2}}\right)(\nu_N-\nu_M). \tag{1}$$

Now, consider the time $t\,(\geq t_Q)$ when car is at S_2 between Q and R. The distance travelled by the car in time t is $\mathrm{PS}_2 = ut$. At this instant, the lines S_2N and S_2M both make angle $(180° - \theta)$ with the velocity vector $\vec{u}$. The component of observer velocity towards the sources N and M is $u_o = -u\cos\theta$. Apply Doppler's effect equation to get

$$\nu'_N = \frac{v+u_o}{v-u_s}\nu_N = \frac{v-u\cos\theta}{v}\nu_N = \left(1-\frac{u}{v}\frac{ut-D}{\sqrt{d^2+(ut-D)^2}}\right)\nu_N,$$
$$\nu'_M = \frac{v+u_o}{v-u_s}\nu_M = \frac{v-u\cos\theta}{v}\nu_M = \left(1-\frac{u}{v}\frac{ut-D}{\sqrt{d^2+(ut-D)^2}}\right)\nu_M.$$

The beat frequency heard by the observer at time $t\,(\geq t_Q)$ is

$$\nu(t) = \nu'_N - \nu'_M = \left(1-\frac{u}{v}\frac{ut-D}{\sqrt{d^2+(ut-D)^2}}\right)(\nu_N-\nu_M). \tag{2}$$

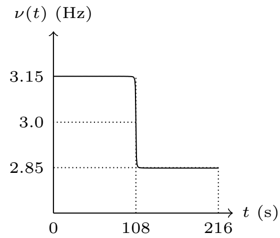
Substitute $t = 0$ and $t = t_Q = D/u$ in equation (1) to get ν_P and ν_Q and substitute $t = t_R = 2D/u$ in equation (2) to get ν_R i.e.,

$$\nu_P = \nu(t=0) = \left(1+\frac{u}{v}\frac{D}{\sqrt{d^2+D^2}}\right)(\nu_N-\nu_M),$$
$$\nu_Q = \nu(t=D/u) = (\nu_N-\nu_M),$$
$$\nu_R = \nu(t=2D/u) = \left(1-\frac{u}{v}\frac{D}{\sqrt{d^2+D^2}}\right)(\nu_N-\nu_M),$$

which gives $\nu_P + \nu_R = 2\nu_Q$. Differentiate equations (1)–(2) w.r.t. time t to get rate of change of beat frequency

$$\frac{\mathrm{d}\nu(t)}{\mathrm{d}t} = -(\nu_N-\nu_M)\frac{u^2}{v}\frac{d^2}{(d^2+(D-ut)^2)^{3/2}}. \tag{3}$$

From equation (3), the slope is negative and its magnitude is maximum when $t = D/u = t_Q$ (denominator is minimum). Thus the rate of change of beat frequency is maximum when car passes through Q. The figure shows that beat frequency is equal to 3.15 Hz at P, it reduces slowly till the car reaches close to Q, at Q the beat frequency reduces sharply, and then it reduces slowly to 2.85 Hz when the car reaches R.



Question 8. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? (2016)

(A) The temperature distribution over the filament is uniform.

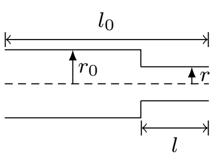
(B) The resistance over small sections of the filament decreases with time.

(C) The filament emits more light at higher band of frequencies before it breaks up.

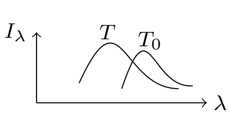
(D) The filament consumes less electrical power towards the end of the life of the bulb.

Solution. The filament breaks up at random location due to non-uniform evaporation of tungsten from the filament. The non-uniform evaporation is caused by the non-uniform temperature distribution over the filament.

Let l_0 be the length and r_0 be the radius of the filament. Let radius of the filament reduces from r_0 to r in a small segment of length $l\,(\ll l_0)$. The resistance of this small segment increases from $\frac{\rho l}{\pi r_0^2}$ to $\frac{\rho l}{\pi r^2}$ ($\because r < r_0$), where ρ is the resistivity of the tungsten.



The heat generated in this small segment in a time interval t, $i^2 t\,\rho l/(\pi r^2)$, is more than the heat generated in other segment of same length but of radius r_0. This heat increases temperature of this segment which further increases its resistance through (i) temperature dependence of resistance (ii) increase in rate of evaporation. Thus, the temperature T of this segment is higher than the temperature T_0 of other segment. A black body at higher temperature emits more radiation at higher band of frequencies (see figure).



Towards the end of life of the bulb, effective resistance R_e of the filament increases (due to decrease in radius caused by evaporation). Thus, the electrical power consumed by the filament, V^2/R_e, decreases.

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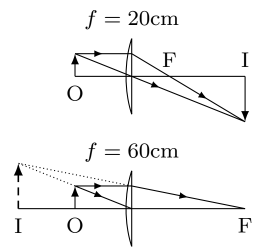
Question 9. A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? (2016)

(A) The refractive index of the lens is 2.5.

(B) The radius of curvature of the convex surface is 45 cm.

(C) The faint image is erect and real.

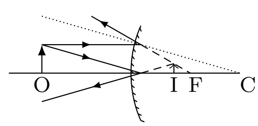
(D) The focal length of the lens is 20 cm.



Solution. For refraction by the plano-convex lens, object distance is $u = -30$ cm. The size of the image is double the size of the object i.e., magnification $m = v/u = \pm 2$, where positive sign is for erect image and negative sign is for inverted image. Thus, the image distance is $v = 60$ cm (inverted image) or $v = -60$ cm (erect image). Apply lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, to get $f = 20$ cm (inverted image) and $f = 60$ cm (erect image).

Let R be the radius of the convex surface of the plano-convex lens which is made of material of refractive index n. Apply lens-maker's formula to get its focal length f as

$$\frac{1}{f} = (n-1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{n-1}{R}. \tag{1}$$



The faint image is formed due to reflection from the convex surface, which acts as a convex mirror. The focal length of this mirror is $f_m = R/2$, object distance is $u = -30$ cm and image distance is $v = 10$ cm (the image by convex mirror is erect, virtual, and is formed towards the right of the pole). Apply the mirror formula, $\frac{1}{f_m} = \frac{1}{v} + \frac{1}{u}$, to get $f_m = 15$ cm. Thus, the radius of curvature of the convex surface is $R = 2f_m = 30$ cm.

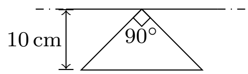
Substitute $R = 30$ cm in equation (1) to get $n = 2.5$ for $f = 20$ cm (inverted image) and $n = 1.5$ for $f = 60$ cm (erect image).

Question 10. A length-scale (l) depends on the permittivity (ϵ) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is(are) dimensionally correct? (2016)

(A) $l = \sqrt{\frac{nq^2}{\epsilon k_B T}}$ (B) $l = \sqrt{\frac{\epsilon k_B T}{nq^2}}$ (C) $l = \sqrt{\frac{q^2}{\epsilon n^{2/3} k_B T}}$ (D) $l = \sqrt{\frac{q^2}{\epsilon n^{1/3} k_B T}}$

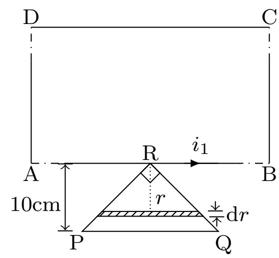
Solution. The dimensions of thermal energy k_BT is $\mathrm{ML^2T^{-2}}$. From Coulomb's law, $F = q_1q_2/(4\pi\epsilon r^2)$, the dimensions of q^2/ϵ is $\mathrm{ML^3T^{-2}}$. The dimensions of number per unit volume n is $\mathrm{L^{-3}}$. Substitute these dimensions in given expressions to get dimensions of $\sqrt{\frac{\epsilon k_BT}{nq^2}}$ and $\sqrt{\frac{q^2}{\epsilon n^{1/3}k_BT}}$ as L.

Question 11. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A/s. Which of the following statement(s) is(are) true? (2016)



(A) The magnitude of induced *emf* in the wire is μ_0/π volt.
(B) If the loop is rotated at a constant angular speed about the wire, an additional *emf* of μ_0/π volt is induced in the wire.
(C) The induced current in the wire is in opposite direction to the current along the hypotenuse.
(D) There is a repulsive force between the wire and the loop.

Solution. We can think of infinite wire as a square loop of infinite dimensions as shown in the figure (note that branches BC, CD, and DA are at infinite distance from the region of interest). Let us denote ABCD as loop 1 and PQR as loop 2. Our interest is to find the mutual inductance between the loop 1 and the loop 2. The flux ϕ_2 through the loop 2 due to the current i_1 in the loop 1 is given by



$$\phi_2 = M_{21}i_1, \tag{1}$$

where M_{21} is the mutual inductance between the loops.

To find the flux ϕ_2, consider a strip of small width dr at a perpendicular distance r from the infinite wire AB (see figure). The area of the strip is $\mathrm{d}A = 2r\mathrm{d}r$. The magnetic field $\vec{B}$ at the strip due to the wire AB and the magnetic flux through the strip are given by

$$\vec{B} = \frac{\mu_0 i_1}{2\pi r}\otimes, \qquad \mathrm{d}\phi_2 = \vec{B}\cdot\mathrm{d}\vec{A} = \frac{\mu_0 i_1}{2\pi r}(2r\mathrm{d}r) = \frac{\mu_0 i_1}{\pi}\,\mathrm{d}r.$$

Integrate $\mathrm{d}\phi_2$ from $r=0$ to $r=0.1$ m to get

$$\phi_2 = \int_0^{0.1}\frac{\mu_0 i_1}{\pi}\,\mathrm{d}r = \frac{0.1\mu_0 i_1}{\pi}. \tag{2}$$

From equations (1) and (2), $M_{21} = 0.1\mu_0/\pi$. The magnetic flux ϕ_1 through loop 1 due to the current i_2 in loop 2 is given by

$$\phi_1 = M_{12}i_2. \tag{3}$$

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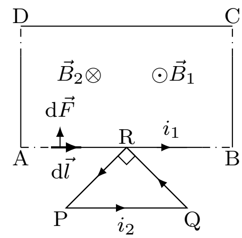
By the *reciprocity theorem of mutual inductance*, $M_{12} = M_{21}$. Thus, we can write equation (3) as

$$\phi_1 = M_{21} i_2 = \frac{0.1\mu_0}{\pi} i_2. \tag{4}$$

By Faraday's law, the induced *emf* in loop 1 is given by

$$e_1 = -\frac{\mathrm{d}\phi_1}{\mathrm{d}t} = -\frac{0.1\mu_0}{\pi}\frac{\mathrm{d}i_2}{\mathrm{d}t}. \qquad \text{(using equation (4))}. \tag{5}$$

Substitute $\mathrm{d}i_2/\mathrm{d}t = 10$ A/s in equation (5) to get the magnitude of induced *emf* in loop 1 (wire AB) as $|e_1| = \mu_0/\pi$.



The current in the loop PQR is counter-clockwise and increases with time. The magnetic field $\vec{B}_2$ due to this loop in the region of the loop ABCD is into the paper and its magnitude increases with time. By Lenz's law, the induced current opposes change in magnetic flux. So, the magnetic field $\vec{B}_1$ due to the induced current in the region of loop ABCD should be out of the paper. This is possible when induced current i_1 is from A to B.

Consider a small element $\mathrm{d}\vec{l}$ on the infinite wire AB. The magnetic field $\vec{B}_2$ due to the loop PQR at the location of this element is into the paper. Thus, magnetic force $\mathrm{d}\vec{F} = i_1\, \mathrm{d}\vec{l} \times \vec{B}_2$ on this element is as shown in the figure (repulsive).

This configuration has rotational symmetry about the wire. Thus, the rotation of the loop PQR at a constant angular speed about the wire will not induce additional *emf* in the wire.

Question 12. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3\, \hat{\imath} + \beta t^2\, \hat{\jmath},$$

where $\alpha = 10/3$ m/s^3, $\beta = 5$ m/s^2 and $m = 0.1$ kg. At $t = 1$ s, which of the following statement(s) is(are) true about the particle? (2016)

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\,\hat{\imath} + 10\,\hat{\jmath})$ m/s.

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -5/3\,\hat{\mathrm{k}}$ N-m-s.

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{\imath} + 2\hat{\jmath})$ N.

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -20/3\,\hat{\mathrm{k}}$ N-m.

Solution. The expressions for the velocity and the acceleration are given by

$$\vec{v}(t) = \mathrm{d}\vec{r}/\mathrm{d}t = 3\alpha t^2\, \hat{\imath} + 2\beta t\, \hat{\jmath}, \tag{1}$$

$$\vec{a}(t) = \mathrm{d}\vec{v}/\mathrm{d}t = 6\alpha t\, \hat{\imath} + 2\beta\, \hat{\jmath}. \tag{2}$$

Substitute $\alpha = 10/3\ \mathrm{m/s^3}$, $\beta = 5\ \mathrm{m/s^2}$, and $t = 1$ s in equation (1) to get $\vec{v} = (10\,\hat{\imath} + 10\,\hat{\jmath})$ m/s. The position vector at $t = 1$ s is $\vec{r} = (10/3\,\hat{\imath} + 5\,\hat{\jmath})$ m. The angular momentum with respect to the origin is given by

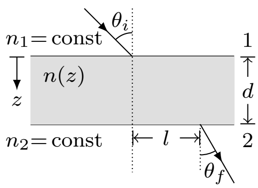
$$\vec{L} = \vec{r} \times (m\vec{v}) = (10/3\,\hat{\imath} + 5\,\hat{\jmath}) \times 0.1(10\,\hat{\imath} + 10\,\hat{\jmath}) = -5/3\,\hat{\mathrm{k}}\ \text{N-m-s}.$$

The acceleration at $t = 1$ s is $\vec{a} = (20\,\hat{\imath} + 10\,\hat{\jmath})\ \mathrm{m/s^2}$. Apply Newton's second law to get force on the particle as $\vec{F} = m\vec{a} = (2\,\hat{\imath} + \hat{\jmath})$ N. The torque with respect to the origin is given by

$$\vec{\tau} = \vec{r} \times \vec{F} = (10/3\,\hat{\imath} + 5\,\hat{\jmath}) \times (2\,\hat{\imath} + \hat{\jmath}) = (-20/3\,\hat{\mathrm{k}})\ \text{N-m}.$$

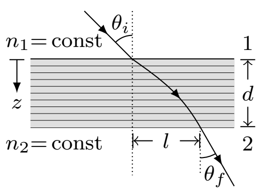
The readers are encouraged to show that $\vec{\tau} = \mathrm{d}\vec{L}/\mathrm{d}t$.

Question 13. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2\,(> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in the medium 2 with refraction angle θ_f with a lateral displacement l. Which of the following statement(s) is(are) true? (2016)



(A) $n_1 \sin\theta_i = n_2 \sin\theta_f$ (B) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$
(C) l is independent of n_2 (D) l is dependent of $n(z)$

Solution. We can approximate the given transparent slab (of varying refractive index) by a large number of parallel slabs of small and equal thicknesses but having different refractive indices. As thickness of each slab is small, we can assume refractive index within a slab as uniform. When a ray passes through a stack of parallel slabs then its angle of emergence θ_f depends on the angle of incidence θ_i and the refractive indices (n_1 and n_2) of the media surrounding the stack i.e.,



$$\sin\theta_f = (n_1/n_2)\sin\theta_i.$$

The readers are encouraged to prove this for a stack of two or three parallel slabs of different refractive indices. Snell's law for the medium of varying refractive index can be written as $n(z)\sin\theta_z = \text{constant}$ at all points on the ray path.

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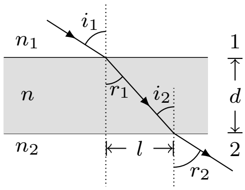
The lateral displacement of a ray by a parallel slab depends on the angle of incidence θ_i, refractive index n_1, slab thickness d and the refractive index of the slab n (it is independent of n_2). Consider the parallel slab shown in the figure. By Snell's law, $n_1 \sin i_1 = n \sin r_1$ and $n \sin i_2 = n_2 \sin r_2$. By geometry, $i_2 = r_1$ and the lateral displacement is given by



$$l = d\,\frac{\sin r_1}{\cos r_1} = \frac{d n_1 \sin i_1}{\sqrt{n^2 - n_1^2 \sin^2 i_1}}.$$

SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks :+3	If only the bubble corresponding to all the correct answer is darkened.
Zero Marks:0	In all other cases.

Question 14. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value of 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C? (2016)

Solution. The power radiated by a metal surface is given by $P = \sigma e A T^4$, where σ is Stefan's constant, e is emissivity of the metal, A is surface area and T is the absolute temperature of the metal. Thus, the powers radiated at temperatures $T_1 = 487\ °\text{C} = 487 + 273 = 760$ K and $T_2 = 2767\ °\text{C} = 3040$ K are

$$P_1 = \sigma e A (760)^4, \qquad P_2 = \sigma e A (3040)^4.$$

The sensor's measurement for these radiated powers are

$$x_1 = \log_2(P_1/P_0) = \log_2\left(\sigma e A (760)^4/P_0\right), \tag{1}$$
$$x_2 = \log_2(P_2/P_0) = \log_2\left(\sigma e A (3040)^4/P_0\right). \tag{2}$$

It is given that $x_1 = 1$. Subtract equation (1) from (2) and simplify to get

$$x_2 = x_1 + \log_2\left(\sigma e A (3040)^4/P_0\right) - \log_2\left(\sigma e A (760)^4/P_0\right)$$
$$= 1 + \log_2\left(\frac{(3040)^4}{(760)^4}\right) = 1 + 4\log_2 4 = 1 + 8 = 9.$$

Question 15. The isotope $^{12}_{5}$B having a mass 12.014 u undergoes β-decay to $^{12}_{6}$C. $^{12}_{6}$C has an excited state of the nucleus ($^{12}_{6}$C*) at 4.041 MeV above its ground state. If $^{12}_{5}$B decays to $^{12}_{6}$C*, the maximum kinetic energy of the β-particle in units of MeV is .... [1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum.] (2016)

Solution. The β-decay is given by the reaction

$$^{12}_{5}\mathrm{B} \to {}^{12}_{6}\mathrm{C} + e^{-1} + \bar{\nu}.$$

The *Q-value* of this reaction is given by

$$Q = \left[m(^{12}_{5}\mathrm{B}) - m(^{12}_{6}\mathrm{C})\right] c^2 = [12.041 - 12.0] \times 931.5 = 13.041\ \mathrm{MeV}.$$

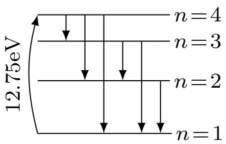
Note that $m(^{12}_{6}\mathrm{C}) = 12$ u by definition of atomic mass unit. The energy $Q = 13.041$ MeV is released in the reaction. Out of this energy, 4.041 MeV is used to excite $^{12}_{6}$C to its excited state $^{12}_{6}$C*. Thus, the kinetic energy available to the β-particle (K_β) and the antineutrino ($K_{\bar{\nu}}$) is $K_\beta + K_{\bar{\nu}} = 13.041 - 4.041 = 9$ MeV. In β-decay, the kinetic energy of the $\bar{\nu}$ can vary from zero to a maximum value. Hence, the maximum kinetic energy of the β-particle is $K_{\beta,\mathrm{max}} = 9$ MeV (when $K_{\bar{\nu}} = 0$).

Question 16. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is .... (2016)

Solution. The energy of the incident photon of wavelength $\lambda = 970$ Å is

$$\Delta E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}} = 12.75\ \mathrm{eV}.$$

Let the incident photon excites hydrogen atom from the ground state ($E_1 = -13.6$ eV) to a state with principal quantum number n ($E_n = -13.6/n^2$ eV). Thus, $E_n = E_1 + \Delta E$ i.e.,



$$-13.6/n^2 = -13.6 + 12.75,$$

which gives $n = 4$. The hydrogen atom can make $^nC_2 = 6$ transitions while returning to ground state (see figure). Thus, the emission spectrum will have six lines.

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Question 17. Consider two solid spheres P and Q each of density 8 g/cm^3 and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 g/cm^3 and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 g/cm^3 and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is .... (2016)

Solution. The terminal velocity of a sphere of radius r and density ρ, immersed in a liquid of density σ and viscosity η, is given by

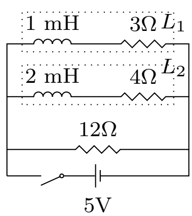
$$v = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}. \tag{1}$$

Substitute the values of given parameters in equation (1) to get

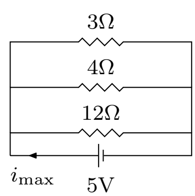
$$\frac{v_P}{v_Q} = \frac{(\rho_P-\sigma_P)\,r_P^2\,\eta_Q}{(\rho_Q-\sigma_Q)\,r_Q^2\,\eta_P} = \frac{(8-0.8)\,(1/2)^2\,(2)}{(8-1.6)\,(0.5/2)^2\,(3)} = 3.$$

Question 18. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current ($I_{\max}/I_{\min}$) drawn from the battery is .... (2016)

Solution. The circuit is shown in the figure. The impedance (effective resistance) of the inductors L_1 and L_2 is very high immediately after the circuit is switched on ($t \to 0^+$). Thus, inductors behave as open circuit elements and entire current flows through the resistor $R = 12\ \Omega$. Thus, by Kirchhoff's law, current through the battery is $i_{\min} = V/R = 5/12$ A.



In steady state ($t \to \infty$), the impedance of the inductors become zero and they behave as resistors of given values (see figure). The effective resistance of the circuit is $R_e = (12\Omega \parallel 4\Omega) \parallel 3\Omega = 3\Omega \parallel 3\Omega = 3/2\ \Omega$. The current through the circuit in steady state is $i_{\max} = V/R_e = 10/3$ A. Thus, $i_{\max}/i_{\min} = (10/3)/(5/12) = 8$.



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